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Spatial Credit Trading in a Shrinking Economy

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Abstract This paper studies a national spatial-credit system that links the permanent removal of vacant floor space in depopulating regions to additional development in high-demand cities. Its central contribution is to separate three failures that require different institutions. First, local majority voting can generate insufficient removal because the decisive resident's exposure to vacant-property damage differs from population-weighted average exposure. Under general population density, this political wedge decomposes into the curvature of spatial exposure and the mean-median difference in residential location. The result remains central under equal incidence, while an extension shows that heterogeneous incidence can change both the identity of the decisive voter and the effect of credit revenue. Second, a competitive spatial-credit market can implement a national net-floor-stock target at least cost across heterogeneous regions. Under complete information and certainty, however, the market is allocatively equivalent to a correctly calibrated development charge and removal subsidy; under uncertainty, the substantive choice is between fixing quantity and fixing price. Third, nominal market clearing need not deliver the physical target when certification overstates additionality or permanence. The model therefore assigns distinct roles to a national cap, competitive trading, a land bank that internalizes local benefits, and a verification system that measures effective contraction. The analysis does not claim a general superiority of markets over subsidies. It shows instead that political choice, interregional allocation, and certification are separate problems and that no single instrument resolves all three.

Keywords: vacant properties; population decline; spatial credits; transferable development rights; majority voting; prices versus quantities; land banks; verification

JEL Classification: D72, H23, H41, R14, R21, R52

1 Introduction

Population decline and durable buildings create a mismatch between people and floor space. Japan provides a particularly visible example. The 2023 Housing and Land Survey reports 9.002 million vacant dwellings and a vacancy rate of 13.8 percent, both record highs. Even after excluding dwellings held for rent or sale and second homes, 3.856 million units remained vacant ([Statistics Bureau of Japan, 2024](#)). Over the same horizon, Japan's population is projected to fall from 126.15 million in 2020 to 87.00 million in 2070 under the medium-fertility and medium-mortality projection ([National Institute of Population and Social Security Research, 2023](#)). Because buildings depreciate and disappear much more slowly than population, unused floor space is not merely a transitional phenomenon. It is a persistent stock problem in a shrinking economy.

Vacancy imposes costs beyond those borne by the owners of abandoned properties. Deteriorated buildings can increase fire and collapse risk, facilitate crime and pests, and lower nearby property values and neighborhood amenity. A shrinking population must also support roads, water and sewer networks, fire protection, and disaster-prevention services over an urbanized footprint that cannot be reduced quickly. Empirical work in the United States and Japan documents meaningful neighborhood spillovers from vacancy, demolition, and rehabilitation ([Rossi-Hansberg, Sarte, and Owens, 2010](#); [Paredes and Skidmore, 2017](#); [Stacy, 2018](#); [Alvayay Torrejón, Paredes, and Skidmore, 2023](#); [Sadayuki, Kanayama, and Arimura, 2019](#)). Japan's 2023 amendment to the Act on Special Measures concerning Vacant Houses likewise extended policy attention from legally designated dangerous buildings to properties at an earlier stage of inadequate management ([Ministry of Land, Infrastructure, Transport and Tourism, 2023](#)).

The regions with the strongest case for removal are often the least able to undertake it. Depopulation increases vacancy, reduces the number of residents over whom removal costs can be spread, and erodes the local tax base. It also creates a political-economy problem. When population is concentrated near the center but the most serious vacancy damage is concentrated toward the fringe, the resident who is decisive under majority voting may face less damage than the average resident. A municipality can then choose too little removal even when the aggregate local benefits exceed the aggregate local costs.

At the same time, some cities continue to face strong demand for residential, commercial, and office floor space. Aggregate population decline therefore does not imply that development should stop everywhere. It implies that the spatial composition of the durable floor stock should change: obsolete floor space should contract in declining regions, while valuable development should remain

possible in high-demand locations. This paper studies a system of tradable spatial credits that links these two margins. Permanent removal in a depopulating region generates certified credits, and developers in a high-demand city surrender credits to obtain additional development rights. A national authority uses the system to control a common measure of net floor stock.

The analysis distinguishes three problems that are frequently conflated. The first is *local political choice*. Under equal sharing of removal costs, local reuse benefits, and credit revenue, the locally efficient amount of removal reflects population-weighted average exposure to vacancy damage, whereas the majority outcome reflects the exposure of the decisive resident. In a centrally concentrated city, these objects need not coincide. The paper shows that the resulting political wedge can be decomposed into two spatial forces: the curvature of the exposure function and the difference between the mean and median residential locations. This decomposition is the paper's principal spatial political-economy result.

The second problem is *interregional allocation*. Conditional on a national quantity target, competitive trading directs more removal toward regions in which an additional effective credit can be supplied at lower net cost. This is the cost-minimization logic of tradable permits (Montgomery, 1972). The paper does not infer from this result that markets are generally superior to taxes and subsidies. Under complete information and certainty, a correctly calibrated development charge and removal subsidy can reproduce the same allocation. Under uncertainty, the substantive distinction is whether policy fixes the aggregate quantity and allows the common price to adjust or fixes the common price and allows the aggregate quantity to adjust. The choice is therefore governed by the prices-versus-quantities logic of Weitzman (1974).

The third problem is *verification*. Credits are valid only to the extent that removal is additional and permanent. A registry may balance perfectly in accounting terms even though the physical contraction is smaller than the certified contraction. The paper shows that over-certification maps directly into a miss of the national net-floor-stock target. Verification is therefore not a secondary administrative detail; it is part of the quantity-control institution itself.

These three problems require different institutional responses. A national cap governs the cross-regional quantity. Competitive trading allocates that quantity across heterogeneous regions. A land bank or another independent local restructuring body is needed when majority voting does not internalize aggregate neighborhood benefits. A certification system is needed to establish additionality, permanence, and the absence of double counting. The paper's contribution is not a general claim that a market dominates a subsidy. It is to show that political choice, market allocation, and verification are distinct margins of failure, and that no single instrument solves all three.

The formal analysis proceeds in two layers. The baseline local model retains equal incidence in order to isolate the spatial exposure mechanism cleanly. An extension then allows removal costs, reuse benefits, and credit revenue to be distributed unevenly across residents. That extension separates the assumptions needed for single-peaked preferences, for a population-weighted median ideal point to be a majority equilibrium, and for that ideal point to coincide with the preferred policy of the resident at the median location. The national model subsequently connects local removal to demand for additional floor space, defines the national quantity, and studies implementation, instrument choice, and certification.

The model is deliberately theoretical. Identifying the exposure function empirically would require geocoded information on vacant properties, physical risk, removal, redevelopment restrictions, and residential population. The framework nevertheless produces measurable objects, including the gap between exposure at the median residential location and population-weighted average exposure, as well as a more general decisive-voter wedge when incidence is heterogeneous.

The remainder of the paper is organized as follows. Section 2 relates the framework to the literature. Section 3 develops the local political economy of removal and introduces the notation and timing. Section 4 derives the baseline spatial political distortion. Section 5 introduces heterogeneous incidence and characterizes the decisive voter. Section 6 develops the national spatial-credit market. Section 7 studies conditional cost efficiency and the choice between price and quantity instruments. Section 8 analyzes verification and target misses. Section 9 draws institutional implications, and Section 10 concludes. The Appendix contains complete proofs, corner solutions, weaker single-crossing conditions, location-specific removal, regional pooling, TIF, and a numerical illustration.

2 Related Literature

2.1 Urban Decline, Durable Housing, and Vacancy Externalities

The spatial structure of the model follows the monocentric-city tradition of [Alonso \(1964\)](#), [Muth \(1969\)](#), and [Fujita \(1989\)](#). The distinctive issue is not the expansion of a city but the slow contraction of a durable stock. [Glaeser and Gyourko \(2005\)](#) show that durable housing causes asymmetric adjustment: positive demand shocks induce construction, whereas negative shocks reduce prices and occupancy while leaving the physical stock in place. [Brueckner and Helsley \(2011\)](#) connect urban sprawl and inner-city blight through development dynamics.

A growing empirical literature measures spillovers from vacancy, demolition, and rehabilitation. Rossi-Hansberg, Sarte, and Owens (2010) estimate housing externalities from urban revitalization. Paredes and Skidmore (2017) estimate the net benefits of demolition in Detroit; Stacy (2018) studies crime effects; vom Hofe, Parent, and Grabill (2019) compare demolition and rehabilitation; and Alvaay Torrejón, Paredes, and Skidmore (2023) estimate impacts on neighboring property values. For Japan, Sadayuki, Kanayama, and Arimura (2019) provide evidence on the externality of vacant houses. The present paper abstracts from empirical estimation and asks how spatially uneven exposure affects local collective choice.

2.2 Majority Voting and Spatial Political Distortion

The voting analysis builds on Black (1948). With a one-dimensional policy and single-peaked preferences, a population-weighted median ideal point is a Condorcet winner. The paper separates two logically distinct steps that are sometimes conflated. Strict concavity generates single-peaked preferences; the median-voter theorem then identifies a population-weighted median ideal point as the majority outcome. Increasing differences are additionally required to order ideal points by residential location and thereby identify the decisive voter with the resident at the population-weighted median location.

The paper's spatial contribution is to compare that resident's exposure with population-weighted average exposure and to decompose the difference into curvature and residential mean-median components. The result gives urban-economic content to the standard median-voter logic and clarifies why central concentration of population can generate under-removal at the periphery.

2.3 Tradable Rights, TDR, and Instrument Choice

Spatial credits draw on the theory of tradable rights. Coase (1960) emphasizes the role of property rights in addressing externalities, and Montgomery (1972) shows that competitive permit trading implements a given aggregate target at least cost. Under certainty, a common permit price and a correctly calibrated common price policy decentralize the same marginal conditions. Under uncertainty, Weitzman (1974) shows that the relative slopes of marginal benefits and marginal costs determine whether fixing quantity or fixing price is preferred.

Transferable Development Rights (TDR) apply tradable rights to land use. Mills (1980) studies TDR markets theoretically, while Carpenter and Heffley (1982) analyze spatial equilibrium with sending and receiving areas. Levinson (1997) shows that interactions with baseline zoning can

increase total development. [Thorsnes and Simons \(1999\)](#) and [McConnell, Walls, and Kopits \(2006\)](#) study market-based preservation and density effects. Spatial credits differ from conventional TDR because issuance requires verified contraction of existing floor space rather than the transfer of unused development potential.

The present paper does not claim that cost information makes permit trading unconditionally superior to a common price instrument. Both can decentralize heterogeneous regional responses. The role of trading is conditional: it provides a least-cost allocation under a quantity target. The aggregate choice between quantity and price remains a Weitzman problem.

2.4 Verification, Land Banks, and Supporting Institutions

Additionality and permanence are central because a spatial credit is valid only if it corresponds to a contraction that would not otherwise have occurred and will not be reversed. The paper treats verification as a distinct institutional problem: inaccurate coefficients create a wedge between the quantity recorded by the registry and the physical net floor stock.

Land banks and related public entities enter for a different reason. A competitive national market can allocate credits efficiently conditional on supply, but local supply may be politically too low when the decisive resident underweights peripheral damage. A land bank that evaluates aggregate neighborhood benefits can therefore complement, rather than substitute for, a national market.

TIF is not a central pillar of the baseline model. Because TIF can finance real public improvements but can also shift revenues across jurisdictions, it is treated as a supporting extension in [Appendix A.10](#). This placement reflects the evidence that TIF designation does not automatically generate net economic growth ([Dye and Merriman, 2000](#)).

2.5 Contribution Relative to the Literature

The literatures above contain the separate ingredients of the model: durable housing, spatial externalities, majority voting, tradable development rights, prices versus quantities, and verification. The contribution is to show that they correspond to different margins of failure in a shrinking economy. Political under-removal is determined inside a region; cost-efficient allocation occurs across regions; and verification determines whether the physical target is met. The framework therefore resists attributing all welfare gains to the creation of a market and instead identifies the institutional combination required for coherent spatial contraction.

3 Local Political Economy of Removal

This section begins with the shrinking region before introducing the national market. The sequence is intentional. Local residents first determine whether vacant floor space is removed and hence whether credits can be supplied at all. We therefore define the inherited stock, the spatial distribution of damage, and the local political decision before connecting the region to developers elsewhere.

3.1 Economic Setting, Units, and Timing

The economy contains J depopulating regions, indexed by $j = 1, \dots, J$, and one high-demand city. Each depopulating region is represented as a linear monocentric area. A location ℓ measures distance from the regional center. The inherited development pattern is summarized by a reference density profile $\rho_j(\ell)$ over the historical urban footprint, while the current population occupies a truncation of that footprint. Vacancy-related damage may vary with location.

To keep population and floor-stock accounting transparent, one resident household is normalized to occupy one standardized unit of floor space. Population, occupied floor space, vacant floor space, and removal are therefore measured in the same standardized physical units. This normalization is not essential; an explicit floor-space demand per household would simply rescale the stock variables.

The within-period sequence is as follows. First, the inherited occupied area, current population density, and regional removal-cost shifter are realized. Second, the region chooses a removal quantity. In the baseline, that quantity is selected by majority voting; in the national-market analysis, a land bank or equivalent institution may choose removal while internalizing aggregate local benefits. Third, a certification authority determines the amount of effective contraction. Finally, credits are traded against demand for additional floor space in the growing city. Regions and residents are price takers in the national credit market.

Table 1 summarizes the principal notation. Every object is also defined at its first use; the table is intended as a guide rather than a substitute for the text.

Table 1: Principal Notation

Symbol	Economic meaning
$j = 1, \dots, J$	Depopulating-region index.

Continued on next page

Table 1 continued

Symbol	Economic meaning
$\ell, L_j, \bar{L}_j$	Distance from the regional center, current occupied boundary, and inherited occupied boundary.
$\rho_j(\ell)$	Current population density at location ℓ .
$N_j, \bar{N}_j, V_j$	Current occupied stock/population, inherited occupied stock, and pre-policy vacant stock.
$x_j, z_j(x_j)$	Nominal removed floor space and the remaining vacancy rate.
$C_j(x_j; \xi_j), \xi_j$	Removal cost and a realized regional cost shifter summarizing physical, legal, and administrative difficulty.
$S_j(x_j)$	Aggregate local benefit from reuse of cleared sites.
$g_j(\ell), \phi_j(z), \gamma_j$	Spatial exposure, the damage response to the vacancy rate, and the damage-scale parameter.
$D_j(x_j)$	Aggregate vacancy-related damage after removal.
$M_j(x_j)$	Aggregate marginal local benefit of removal, including reuse and avoided damage.
$A_j(x_j; \xi_j)$	Marginal net removal cost after subtracting locally internalized marginal benefits.
$\ell_j^m, \bar{\ell}_j$	Population-weighted median and mean residential locations.
x_j^T, x_j^M	Locally efficient and majority-voting removal quantities in the equal-incidence baseline.
$\tau_j(\ell), s_j(\ell), r_j(\ell)$	Resident shares of removal cost, reuse benefit, and credit revenue.
$y, m, F_0(y; m)$	Additional floor space in the growing city, its demand shifter, and net local development benefit.
η, κ_j	Technical conversion factors from development and removal into a common floor-stock unit.
$\omega_j, \hat{\omega}_j$	True and certified effective-contraction coefficients.
$Q, \bar{Q}, H(Q)$	Realized national net floor stock, its policy target, and the cross-regional external cost.
q, q^*	Competitive credit price and the first-best shadow value of the national quantity.

Continued on next page

Table 1 continued

Symbol	Economic meaning
A_0, R^G	Baseline development allowances and credits acquired and retired by the government.

3.2 Depopulating Regions and Vacancy

Region j inherited an occupied urban area $[0, \bar{L}_j]$ from an earlier period, but the current occupied area is only $[0, L_j]$, where $L_j < \bar{L}_j$. The reference density profile is $\rho_j(\ell) > 0$ on the historical footprint, and the current city is represented as its truncation at L_j . Using the one-household–one-standardized-unit normalization introduced above, inherited and current occupied floor stocks are

$$\bar{N}_j = \int_0^{\bar{L}_j} \rho_j(\ell) d\ell, \quad N_j = \int_0^{L_j} \rho_j(\ell) d\ell. \quad (3.1)$$

The first integral is the stock supported by the earlier population distribution; the second is the currently occupied stock. Their difference,

$$V_j \equiv \bar{N}_j - N_j > 0, \quad (3.2)$$

is the pre-policy stock of vacant standardized floor units.

The regional policy variable is $x_j \in [0, V_j]$, the amount of vacant floor space permanently removed. The baseline assumes proportional removal: removal reduces the aggregate vacancy rate without changing the relative spatial exposure profile $g_j(\ell)$. The remaining vacancy rate is therefore

$$z_j(x_j) \equiv \frac{V_j - x_j}{\bar{N}_j}. \quad (3.3)$$

This assumption isolates the effect of the amount removed. Appendix A.6 relaxes it by considering high-damage-first removal, fringe-first removal, and a location-specific removal vector.

Removal entails the aggregate resource cost $C_j(x_j; \xi_j)$. The shifter ξ_j summarizes realized physical and institutional conditions such as building type, asbestos treatment, fragmented ownership, access constraints, and the administrative difficulty of compulsory action. It is known locally when removal is chosen. The full-information benchmark later assumes that the national planner also observes it, whereas the uncertainty analysis treats it as not fully known when the policy instrument

is set. Marginal and incremental costs are positive:

$$C_{jx} > 0, \quad C_{jxx} > 0. \quad (3.4)$$

Cleared sites may be converted to green space, disaster-mitigation space, public space, farmland, or another locally valuable use. Let $S_j(x_j)$ denote the resulting aggregate local benefit, with

$$S'_j \geq 0, \quad S''_j \leq 0. \quad (3.5)$$

The function S_j records positive benefits generated by the post-removal use of the site. It does not include the avoided harm from the dangerous or abandoned structure itself; that effect is represented separately below. The distinction prevents the same local benefit from being counted twice.

3.3 Spatial Exposure and Aggregate Local Damage

Vacancy damage has both a spatial and an aggregate component. A resident at location $\ell \in [0, L_j]$ experiences the loss

$$e_j(\ell, x_j) = \gamma_j g_j(\ell) \phi_j(z_j(x_j)). \quad (3.6)$$

The scale parameter $\gamma_j > 0$ converts exposure into utility loss. The function $g_j(\ell) \geq 0$ measures the resident's spatial exposure to the region's vacant stock, while $\phi_j(z)$ describes how the common vacancy rate amplifies that exposure. This multiplicative formulation is a tractable reduced form: the location term identifies who is most affected, and the vacancy-rate term identifies how the severity of damage changes when the aggregate stock is reduced. We assume

$$g'_j(\ell) > 0, \quad \phi'_j(z) > 0, \quad \phi''_j(z) \geq 0. \quad (3.7)$$

Residents farther from the center are more exposed in the baseline spatial ordering. A higher vacancy rate raises damage, and convexity means that the damage from an already high vacancy rate increases more than proportionally. Since removal lowers z_j , this convexity makes the marginal damage reduction from removal decline as more units are removed.

To compare individual political preferences with aggregate welfare, define population-weighted average exposure by

$$\bar{g}_j \equiv \frac{1}{N_j} \int_0^{L_j} \rho_j(\ell) g_j(\ell) d\ell. \quad (3.8)$$

Aggregate local damage is

$$D_j(x_j) = \gamma_j N_j \bar{g}_j \phi_j(z_j(x_j)). \quad (3.9)$$

The aggregate marginal local benefit of removal is

$$M_j(x_j) \equiv S'_j(x_j) - D'_j(x_j) = S'_j(x_j) + \frac{\gamma_j N_j}{\bar{N}_j} \bar{g}_j \phi'_j(z_j(x_j)). \quad (3.10)$$

The object that will later govern regional credit supply is the marginal resource cost net of all local marginal benefits:

$$A_j(x_j; \xi_j) \equiv C_{jx}(x_j; \xi_j) - M_j(x_j). \quad (3.11)$$

Thus A_j is not the engineering cost of demolition alone. It is the additional social resource cost of obtaining one more unit of removal after subtracting the reuse benefit and the locally internalized reduction in vacancy damage. Under equations (3.4)–(3.7),

$$A_{jx} = C_{jxx} - S''_j + \frac{\gamma_j N_j}{\bar{N}_j^2} \bar{g}_j \phi''_j > 0. \quad (3.12)$$

The increasing function A_j plays two roles. Locally, its zero identifies the efficient amount of removal before credit revenue. Nationally, its value per effective credit is the relevant marginal supply cost. Appendix A.1 derives equations (3.9)–(3.12) from individual damage and verifies the concavity properties used below.

3.4 Local Social Choice and Equal Incidence

The baseline political model deliberately maintains equal incidence so that the only disagreement among residents comes from spatial exposure. Every resident pays the same per-capita share of removal cost and receives the same per-capita share of local reuse benefits. The national credit market has not yet been introduced, so credit revenue is set to zero. Up to terms that do not vary with x_j , the utility of a resident at location ℓ is

$$u_j^0(\ell, x_j) = \bar{u}_j(\ell) + \frac{S_j(x_j) - C_j(x_j; \xi_j)}{N_j} - \gamma_j g_j(\ell) \phi_j(z_j(x_j)). \quad (3.13)$$

Population aggregation of equation (3.13) yields local welfare. Because costs and reuse benefits are shared equally while damage is integrated over the population, aggregate local welfare is

$$W_j^T(x_j) = S_j(x_j) - C_j(x_j; \xi_j) - D_j(x_j). \quad (3.14)$$

Let x_j^T denote the quantity that a local decision maker would choose if it internalized every resident's utility. It maximizes equation (3.14). At an interior solution, the marginal net removal cost defined in equation (3.11) is zero:

$$A_j(x_j^T; \xi_j) = 0, \quad (3.15)$$

or equivalently

$$C_{jx}(x_j^T; \xi_j) = S'_j(x_j^T) + \frac{\gamma_j N_j}{\bar{N}_j} \bar{g}_j \phi'_j(z_j(x_j^T)). \quad (3.16)$$

We next characterize majority voting. The first step is to establish that every resident has a unique most-preferred removal quantity. Differentiating individual utility twice gives

$$u_{jxx}^0(\ell, x_j) = \frac{S''_j(x_j) - C_{jxx}(x_j; \xi_j)}{N_j} - \frac{\gamma_j}{\bar{N}_j^2} g_j(\ell) \phi''_j(z_j(x_j)) < 0. \quad (3.17)$$

Preferences over the one-dimensional policy x_j are therefore strictly single peaked. The second step is to order residents' ideal points by location. In the equal-incidence baseline, the cross derivative is positive:

$$u_{jx\ell}^0(\ell, x_j) = \frac{\gamma_j}{\bar{N}_j} g'_j(\ell) \phi'_j(z_j(x_j)) > 0. \quad (3.18)$$

Consequently, residents farther from the center weakly prefer more removal. This location ordering allows the median ideal point to be identified with a physical residential location.

Define the population-weighted median residential location ℓ_j^m by

$$\int_0^{\ell_j^m} \rho_j(\ell) d\ell = \frac{N_j}{2}. \quad (3.19)$$

Because ideal points are ordered by location, the population-weighted median ideal point is the ideal point of the resident at ℓ_j^m . By the median-voter theorem, this quantity is the majority outcome, denoted x_j^M . At an interior solution,

$$C_{jx}(x_j^M; \xi_j) = S'_j(x_j^M) + \frac{\gamma_j N_j}{\bar{N}_j} g_j(\ell_j^m) \phi'_j(z_j(x_j^M)). \quad (3.20)$$

Appendix A.2 supplies the complete proof. It also treats boundary choices and derives conditions under which the majority chooses no removal even though the local social optimum is strictly positive.

4 Spatial Exposure and Political Distortions

The preceding section derived two removal conditions from the same primitives. The local optimum values the damage reduction received by the entire population, whereas majority voting values the damage reduction perceived by the decisive resident. This section compares the two and then gives the resulting wedge a spatial interpretation that can be taken to data.

4.1 General Density and Majority Voting

The local social optimum in equation (3.16) weights exposure by $\bar{g}_j$. The majority outcome in equation (3.20) weights exposure by $g_j(\ell_j^m)$. Because the remaining parts of the two marginal conditions are identical, their comparison is governed by this exposure gap.

Proposition 4.1 (General Density and Voting Distortions). *Suppose the local social optimum and the majority outcome are interior. Then*

$$g_j(\ell_j^m) < \bar{g}_j \implies x_j^M < x_j^T, \quad (4.1)$$

$$g_j(\ell_j^m) = \bar{g}_j \implies x_j^M = x_j^T, \quad (4.2)$$

$$g_j(\ell_j^m) > \bar{g}_j \implies x_j^M > x_j^T. \quad (4.3)$$

With corner solutions, the corresponding weak inequalities hold. In particular, it is possible that $x_j^M = 0 < x_j^T$.

The proof uses the fact that A_j is strictly increasing. Replacing average exposure in the social condition with the decisive resident's exposure therefore shifts the unique solution in the same direction as the exposure difference. Proposition 4.1 isolates a political distortion that is spatial rather than fiscal. Under equal incidence, all residents face the same per-capita cost and receive the same per-capita reuse benefit. The disagreement arises because they experience different local damage. A centrally located majority may therefore choose no removal even when aggregate local welfare is increasing at the origin. Appendix A.2 proves the interior and corner statements and makes this zero-removal case explicit.

4.2 Curvature and the Mean-Median Structure of Residence

The exposure comparison in Proposition 4.1 is economically transparent but still reduced form. To identify its spatial sources, define the population-weighted mean residential location

$$\bar{\ell}_j \equiv \frac{1}{N_j} \int_0^{L_j} \ell \rho_j(\ell) d\ell. \quad (4.4)$$

Adding and subtracting exposure evaluated at this mean location yields the following exact identity:

$$\begin{aligned} \bar{g}_j - g_j(\ell_j^m) &= \underbrace{\left[\bar{g}_j - g_j(\bar{\ell}_j) \right]}_{\text{exposure-curvature effect}} \\ &+ \underbrace{\left[g_j(\bar{\ell}_j) - g_j(\ell_j^m) \right]}_{\text{residential mean-median effect}}. \end{aligned} \quad (4.5)$$

Proposition 4.2 (Spatial Decomposition of the Political Distortion). *If g_j is convex and $\ell_j^m \leq \bar{\ell}_j$, then*

$$\bar{g}_j \geq g_j(\ell_j^m). \quad (4.6)$$

If either g_j is strictly convex over a set of positive population measure or $\ell_j^m < \bar{\ell}_j$, then the inequality is strict and, for interior choices, $x_j^M < x_j^T$.

The first component in equation (4.5) compares average exposure with exposure at the average location. It is nonnegative under convexity by Jensen's inequality. The second compares the average and median locations and is nonnegative when the residential distribution has a peripheral tail. Thus the first component is positive when exposure rises convexly toward the vacant fringe. The second is positive when the residential distribution has a peripheral tail, so that the mean resident is farther from the center than the median resident. These are distinct mechanisms. Either can generate political under-removal; together they reinforce it.

Corollary 4.3 (Constant Density Benchmark). *If $\rho_j(\ell) = \rho_{0j}$ and $g_j(\ell) = \ell/L_j$, then*

$$\ell_j^m = \bar{\ell}_j = \frac{L_j}{2}, \quad \bar{g}_j = g_j(\ell_j^m) = \frac{1}{2}, \quad (4.7)$$

so $x_j^M = x_j^T$.

Corollary 4.4 (Centrally Concentrated Exponential Density). *If $\rho_j(\ell) = \rho_{0j}e^{-\lambda_{dj}\ell}$, $\lambda_{dj} > 0$, and*

$g_j(\ell) = \ell/L_j$, then

$$\ell_j^m = -\frac{1}{\lambda_{dj}} \log\left(\frac{1 + e^{-\lambda_{dj}L_j}}{2}\right) < \frac{1}{\lambda_{dj}} - \frac{L_j e^{-\lambda_{dj}L_j}}{1 - e^{-\lambda_{dj}L_j}} = \bar{\ell}_j, \quad (4.8)$$

and hence $x_j^M < x_j^T$ for interior choices. As $\lambda_{dj}L_j \rightarrow \infty$, $\ell_j^m \rightarrow (\log 2)/\lambda_{dj}$ while $\bar{\ell}_j \rightarrow 1/\lambda_{dj}$, so the gap does not vanish under strong central concentration.

Appendix A.3 proves Proposition 4.2 and both density corollaries. It also derives the limiting expressions in Corollary 4.4; Appendix A.2 contains the underlying voting comparison.

5 Distributional Incidence and the Decisive Voter

The equal-incidence baseline is the cleanest environment for the paper's main spatial result, but actual removal programs need not distribute costs and benefits uniformly. Property owners, taxpayers, nearby residents, and recipients of credit revenue may be different groups. This section therefore introduces heterogeneous incidence as an extension rather than as a replacement for the baseline. It has two purposes: to state the majority-voting logic in the correct order, and to identify when credit revenue changes the political wedge.

5.1 Heterogeneous Incidence and Aggregation

Anticipating the national market developed in Section 6, take the common credit price q as given. A resident at location ℓ in region j then has utility

$$\begin{aligned} u_j(\ell, x_j; q) = & \bar{u}_j(\ell) - \tau_j(\ell)C_j(x_j; \xi_j) + s_j(\ell)S_j(x_j) \\ & + r_j(\ell)q\kappa_j\omega_jx_j - \gamma_jg_j(\ell)\phi_j(z_j(x_j)). \end{aligned} \quad (5.1)$$

Here $\tau_j(\ell)$ is the resident's share of removal cost, $s_j(\ell)$ is the share of local reuse benefits, and $r_j(\ell)$ is the share of credit revenue. Assume

$$\tau_j(\ell) > 0, \quad s_j(\ell) \geq 0, \quad r_j(\ell) \geq 0, \quad (5.2)$$

and impose the adding-up conditions

$$\int_0^{L_j} \rho_j(\ell)\tau_j(\ell) d\ell = 1, \quad (5.3)$$

$$\int_0^{L_j} \rho_j(\ell) s_j(\ell) d\ell = 1, \quad \int_0^{L_j} \rho_j(\ell) r_j(\ell) d\ell = 1. \quad (5.4)$$

The equal-incidence baseline is the special case

$$\tau_j(\ell) = s_j(\ell) = r_j(\ell) = \frac{1}{N_j}. \quad (5.5)$$

These restrictions are accounting conditions, not assumptions of equal incidence. They ensure that one unit of aggregate cost, benefit, or revenue is neither omitted nor counted more than once when individual utilities are added. Population aggregation of equation (5.1) therefore gives, up to invariant terms,

$$W_j(x_j; q) = S_j(x_j) - C_j(x_j; \xi_j) + q\kappa_j\omega_j x_j - D_j(x_j). \quad (5.6)$$

Thus heterogeneous incidence redistributes a given total burden or benefit without changing its aggregate amount. Appendix A.1 records the aggregation calculation for the baseline damage term; the same adding-up logic applies here.

Throughout this section, the region and its residents take the national credit price q as given. The price-taking assumption is important. If a large region internalized $q = q(x_j)$, credit revenue would be nonlinear in removal and both first- and second-order conditions would contain q' and q'' . Strategic regional market power is outside the present model.

5.2 Single-Peakedness and the Population-Weighted Median Ideal Point

Differentiating equation (5.1) twice yields

$$u_{jxx}(\ell, x_j; q) = -\tau_j(\ell)C_{jxx}(x_j; \xi_j) + s_j(\ell)S_j''(x_j) - \frac{\gamma_j}{\bar{N}_j^2}g_j(\ell)\phi_j''(z_j(x_j)). \quad (5.7)$$

Under equations (3.4), (3.5), (3.7), and (5.2), $u_{jxx} < 0$. Each resident therefore has a unique ideal point and single-peaked preferences. The credit-revenue term does not enter the second derivative because it is linear in x_j and the region takes q as given.

Let the unique ideal removal quantity of resident ℓ be

$$x_j^I(\ell; q) \equiv \arg \max_{x_j \in [0, V_j]} u_j(\ell, x_j; q). \quad (5.8)$$

The population-weighted distribution of ideal points is

$$F_j^I(x; q) \equiv \frac{1}{N_j} \int_0^{L_j} \rho_j(\ell) \mathbf{1}\{x_j^I(\ell; q) \leq x\} d\ell. \quad (5.9)$$

Since a location with more residents carries more votes, the relevant median is the median of this population-weighted distribution, not the geometric median of the set of locations. A population-weighted median ideal point $x_j^{\text{dec}}(q)$ satisfies

$$F_j^I(x_j^{\text{dec}}; q) \geq \frac{1}{2}, \quad 1 - F_j^I(x_j^{\text{dec}} -; q) \geq \frac{1}{2}. \quad (5.10)$$

5.3 Single Crossing and Identification by Location

The cross derivative is

$$\begin{aligned} u_{jx\ell}(\ell, x_j; q) = & -\tau_j'(\ell)C_{jx}(x_j; \xi_j) + s_j'(\ell)S_j'(x_j) + r_j'(\ell)q\kappa_j\omega_j \\ & + \frac{\gamma_j}{N_j}g_j'(\ell)\phi_j'(z_j(x_j)). \end{aligned} \quad (5.11)$$

Increasing differences, $u_{jx\ell} \geq 0$ for all (ℓ, x_j) , is a transparent sufficient condition for ideal removal to be nondecreasing in location, including at corners. For an interior ideal point, the weaker condition need hold only along the ideal-point locus because

$$\frac{dx_j^I(\ell; q)}{d\ell} = -\frac{u_{jx\ell}(\ell, x_j^I(\ell; q); q)}{u_{jxx}(\ell, x_j^I(\ell; q); q)}. \quad (5.12)$$

Appendix A.4 proves the weighted-median result without assuming that ideal points are ordered by location. It then shows that, for interior ideal points, the global increasing-differences condition can be weakened to a condition evaluated only along the ideal-point locus.

Proposition 5.1 (Distributional Incidence and the Decisive Voter). *Under strict concavity, each resident has single-peaked preferences over x_j , and every population-weighted median of the ideal-point distribution in equation (5.9) is a majority-voting equilibrium. Single crossing is not required for this result. If, in addition, $u_{jx\ell} \geq 0$ on $[0, L_j] \times [0, V_j]$, ideal points are nondecreasing in location. When the weighted medians are unique,*

$$x_j^{\text{dec}}(q) = x_j^I(\ell_j^m; q), \quad (5.13)$$

where ℓ_j^m is the population-weighted median residential location defined in equation (3.19).

The logical order is therefore

$$\text{strict concavity} \implies \text{single-peaked preferences} \implies \text{weighted median ideal point} \quad (5.14)$$

followed, only under single crossing, by

$$\text{weighted median ideal point} \implies \text{ideal point of the median-location resident.} \quad (5.15)$$

This separation matters once cost burdens, reuse benefits, and credit revenue vary across space. Single-peakedness is sufficient for a median ideal point to defeat alternatives in pairwise voting; single crossing is needed only to attach that ideal point to the resident at a particular location.

5.4 Conditional Neutrality of Credit Revenue

Let ℓ_j^d denote the decisive resident when ideal points are monotone. The aggregate marginal payoff from removal, including credit revenue at a given price, is

$$W'_j(x_j; q) = -C_{jx} + S'_j + q\kappa_j\omega_j + \frac{\gamma_j N_j}{\bar{N}_j} \bar{g}_j \phi'_j. \quad (5.16)$$

Arguments of derivatives are suppressed in this subsection. Scale the decisive resident's marginal utility by N_j . The difference between the aggregate and decisive marginal evaluations is

$$\begin{aligned} \Delta_j(x_j; q) &\equiv W'_j(x_j; q) - N_j u_{jx}(\ell_j^d, x_j; q) \\ &= \left[N_j \tau_j(\ell_j^d) - 1 \right] C_{jx} + \left[1 - N_j s_j(\ell_j^d) \right] S'_j \\ &\quad + \left[1 - N_j r_j(\ell_j^d) \right] q\kappa_j\omega_j \\ &\quad + \frac{\gamma_j N_j}{\bar{N}_j} \left[\bar{g}_j - g_j(\ell_j^d) \right] \phi'_j. \end{aligned} \quad (5.17)$$

A positive value means that aggregate welfare places a larger marginal value on removal than the decisive resident does.

Proposition 5.2 (Global Neutrality under Equal Incidence and Local Effects under Heterogeneous

Incidence). Under equal incidence, equation (5.17) reduces to

$$\Delta_j(x_j; q) = \frac{\gamma_j N_j}{\bar{N}_j} \left[\bar{g}_j - g_j(\ell_j^m) \right] \phi'_j, \quad (5.18)$$

which is independent of q . Hence equally distributed credit revenue shifts the aggregate and decisive marginal incentives by the same amount and does not alter the exposure-based political wedge. Under equal incidence, this neutrality is global: the credit price does not affect the ordering of ideal points, and the decisive location remains the population-weighted median residential location. Under heterogeneous incidence, by contrast, the following derivative is a local comparative-static result on any price interval over which the relevant ordering of ideal points and the identity of the decisive voter remain unchanged:

$$\frac{\partial \Delta_j}{\partial q} = \left[1 - N_j r_j(\ell_j^d) \right] \kappa_j \omega_j. \quad (5.19)$$

Credit revenue concentrated on the decisive resident, $r_j(\ell_j^d) > 1/N_j$, reduces the wedge and may eliminate or reverse under-removal; revenue concentrated away from that resident increases the wedge. If a change in q alters single crossing, the ordering of ideal points, the set of tied median ideal points, or the relevant corner structure, equation (5.19) cannot be applied as a global comparative static.

Proposition 5.2 replaces the unconditional statement that credit revenue never changes political distortion. Neutrality is a property of equal incidence, not of credit revenue itself. Appendix A.5 derives equation (5.17), proves the proposition, and discusses the qualification that a sufficiently large change in q may change the identity of the decisive voter.

6 The National Spatial-Credit Market

We now connect local removal to development elsewhere. The construction proceeds in four steps. First, we derive demand for additional floor space in the growing city. Second, we convert development and permanent removal into a common physical unit. Third, we distinguish public costs already internalized locally from costs shifted across regions. Finally, we formulate the planner's benchmark and the decentralized market. This order keeps the national quantity from appearing as an unexplained institutional constraint.

6.1 Demand for Additional Floor Space

A high-demand city uses credits to obtain additional development rights. The model does not assume that every person leaving a shrinking region moves to this city within the same period. Instead, let m be a predetermined demand shifter that summarizes household and firm demand for additional floor space. It can reflect prior population relocation, firm entry, and agglomeration forces, but it does not respond to current removal x_j in the one-period model.

Let $P(s; m)$ be inverse demand for the s th unit of additional floor space, with $P_s < 0$ and $P_m > 0$. Let $K(y)$ be construction cost and let $G(y, m)$ include only costs internal to the growing city, such as local congestion, transport and utility-capacity burdens, and local disaster exposure. The area under inverse demand is gross willingness to pay. After subtracting construction and the costs borne within the growing city, net local development benefit is

$$F_0(y; m) \equiv \int_0^y P(s; m) ds - K(y) - G(y, m), \quad (6.1)$$

where $F_{0y} > 0$ for small y , $F_{0yy} < 0$, and $F_{0ym} > 0$ over the relevant range. Thus additional development is initially valuable, its marginal value declines with scale, and stronger underlying demand shifts the marginal-benefit schedule outward.

6.2 Effective Contraction and the National Net Floor Stock

Nominal demolition is not necessarily equivalent to a permanent policy-induced contraction. Let $a_j \in [0, 1]$ be the share of nominal removal that would not have occurred without the credit program, and let $p_j \in [0, 1]$ denote the share of additional removals that remains permanent rather than being rebuilt later. Define the true effective coefficient

$$\omega_j \equiv a_j p_j \in [0, 1]. \quad (6.2)$$

Because p_j is defined conditional on removal being additional, $\omega_j = a_j p_j$ is the joint effective share; no statistical independence between additionality and permanence is assumed. Sections 6 and 7 maintain perfect certification, $\hat{\omega}_j = \omega_j$. Section 8 relaxes this assumption. Each unit of nominal removal x_j therefore produces $\omega_j x_j$ units of effective contraction.

Let $\eta > 0$ convert one unit of additional floor area in the growing city into a common physical floor-stock unit, and let $\kappa_j > 0$ convert one unit of removed floor area in region j into the same

unit. These are technical conversion coefficients. They do not include land-price gains, green-space benefits, disaster-mitigation benefits, or local regeneration benefits. Avoided dangerous-building damage is valued through $-D'_j(x_j)$, and positive reuse benefits are valued through $S'_j(x_j)$. Keeping those benefits outside κ_j preserves the physical meaning of the national quantity and avoids double counting.

Measured relative to the policy baseline, the national net floor stock is the common-unit expansion of floor space in the growing city minus effective permanent contraction in all shrinking regions:

$$Q \equiv \eta y - \sum_{j=1}^J \kappa_j \omega_j x_j. \quad (6.3)$$

A larger Q therefore means that additional floor space exceeds effective permanent contraction by a larger amount. The equation is a physical accounting definition, not a welfare index: local land-value gains, safety benefits, and reuse benefits remain in F_0 , D_j , and S_j rather than being folded into η or κ_j .

6.3 Local and Cross-Regional Public Costs

The national government needs a reason to regulate Q rather than leaving every local effect to local policy. To isolate that reason and avoid double counting, decompose the public burden associated with removal and development conceptually as

$$\sum_{j=1}^J B_j^L(x_j) + B_G^L(y) + B^T(Q). \quad (6.4)$$

$B_j^L(x_j)$ denotes costs and benefits borne within shrinking region j and is already reflected in C_j , D_j , and S_j . $B_G^L(y)$ denotes costs borne within the growing city and is already reflected in $G(y, m)$. The remaining term $B^T(Q)$ captures only costs shifted across regions through intergovernmental transfers, national subsidies, or common taxes when the national net floor stock is excessive relative to population. Define

$$H(Q) \equiv B^T(Q), \quad (6.5)$$

with

$$H'(Q) > 0, \quad H''(Q) \geq 0. \quad (6.6)$$

These conditions are imposed over the relevant policy range in which the national net floor stock exceeds its population-consistent benchmark; they are not intended as global restrictions on all conceivable values of Q . This definition makes H the nationally externalized fiscal and land-use burden and excludes local effects already counted elsewhere. The most direct interpretation is the portion of infrastructure and spatial-maintenance costs financed through intergovernmental transfers, national subsidies, or common taxation rather than by the jurisdiction generating the burden.

The scalar Q is an explicit aggregation assumption. It requires location-specific floor changes to be convertible into a common certified unit through η , κ_j , and ω_j . If several non-substitutable spatial objectives were relevant, the proper institution would use differentiated credit classes or a vector of targets rather than a single national market.

6.4 The Full-Information Benchmark

The full-information planner provides the welfare benchmark. It observes each realized cost shifter ξ_j , the growing-city demand shifter m , and the true effectiveness coefficients. It chooses $\mathbf{x} = (x_1, \dots, x_J)$ and y to maximize

$$W(\mathbf{x}, y; \xi) = \sum_{j=1}^J [S_j(x_j) - C_j(x_j; \xi_j) - D_j(x_j)] + F_0(y; m) - H(Q). \quad (6.7)$$

At an interior first-best,

$$A_j(x_j^*; \xi_j) = \kappa_j \omega_j q^*, \quad j = 1, \dots, J, \quad (6.8)$$

$$F_{0y}(y^*; m) = \eta q^*, \quad (6.9)$$

where

$$q^* \equiv H'(Q^*), \quad Q^* = \eta y^* - \sum_{j=1}^J \kappa_j \omega_j x_j^*. \quad (6.10)$$

The value q^* is the shadow price of one common unit of the national net floor stock. Equations (6.8)–(6.10) show that it simultaneously values an extra effective unit of removal, an extra unit of credit demand from development, and the marginal cross-regional burden. Appendix A.7 derives these conditions and treats boundary cases.

6.5 Market Accounting

We next translate the target into market accounting. Let A_0 be a stock of baseline development allowances issued by the national government. These allowances permit some development without contemporaneous regional removal. Let R^G be credits acquired by the government and permanently retired. Developers must surrender credits for additional floor space, while regional removal creates certified credits. The registry balance is

$$\eta y + R^G = A_0 + \sum_{j=1}^J \kappa_j \omega_j x_j. \quad (6.11)$$

Rearranging the registry balance gives

$$Q = A_0 - R^G \equiv \bar{Q}. \quad (6.12)$$

The difference $A_0 - R^G$ is the book target for the national net floor stock. The government can therefore implement a positive, zero, or negative target by choosing baseline allowances and retirement. This formulation also makes clear that the credit market need not be exactly self-financing between developers and removing regions when $\bar{Q} \neq 0$.

Given the target, the market price allocates supply and demand. Taking the national price q as given, a land bank in shrinking region j chooses

$$\max_{0 \leq x_j \leq V_j} \{S_j(x_j) - C_j(x_j; \xi_j) - D_j(x_j) + q \kappa_j \omega_j x_j\}. \quad (6.13)$$

At an interior solution, this problem gives the regional credit-supply condition

$$A_j(x_j; \xi_j) = q \kappa_j \omega_j \quad (6.14)$$

for an interior supplier. Because Sections 6 and 7 maintain perfect certification, equation (6.14) is written with the true coefficient ω_j ; Section 8 distinguishes it from the certified coefficient $\hat{\omega}_j$. A developer in the growing city pays $q\eta$ for each unit of additional floor space and chooses

$$\max_{y \geq 0} \{F_0(y; m) - q\eta y\}, \quad (6.15)$$

with interior demand condition

$$F_{0y}(y; m) = q\eta. \quad (6.16)$$

When the target is correctly chosen, the competitive equilibrium price equals the planner's shadow value, $q = q^*$. The land-bank assumption is substantive: it means that local benefits in $S_j - D_j$ are internalized when credits are supplied. Section 9 explains why ordinary majority choice need not produce this supply.

7 Conditional Cost Efficiency and Instrument Choice

This section makes explicit the hierarchy among full-information implementation, conditional cost efficiency, and instrument choice under uncertainty. The distinction is important for interpreting what the credit market does and does not accomplish. Full-information implementation asks whether the market can decentralize a known optimum. Conditional efficiency asks how a given aggregate target is distributed across regions. Instrument choice asks whether the aggregate target should be fixed in advance when costs are uncertain. The three claims are related but not interchangeable.

7.1 Full-Information Implementation and Price Equivalence

Suppose first that the national authority observes all realized costs and demand. It can then calculate the first-best value Q^* rather than merely an expected target, and can set $\bar{Q} = Q^*$ through A_0 and R^G . Equations (6.14), (6.16), and (6.11) then reproduce the planner conditions.

This implementation result does not establish a general superiority of trading. A matched price policy can reproduce the same allocation. Charge developers $q^*\eta$ per unit of additional floor space and subsidize effective removal in region j by $q^*\kappa_j\omega_j$ per unit. Development-charge revenue minus removal-subsidy expenditure is

$$q^*\eta y^* - q^* \sum_{j=1}^J \kappa_j \omega_j x_j^* = q^* Q^* = q^* \bar{Q}. \quad (7.1)$$

If $\bar{Q} > 0$, charge revenue exceeds subsidy expenditure; if $\bar{Q} < 0$, subsidy expenditure exceeds charge revenue; and if $\bar{Q} = 0$, the program balances. The difference is returned or financed through a lump-sum transfer or tax and creates no additional marginal distortion.

7.2 A Given Aggregate Target

Now hold the aggregate amount of effective contraction fixed and ask only how it should be distributed. Let

$$X \equiv \sum_{j=1}^J \kappa_j \omega_j x_j. \quad (7.2)$$

For a prescribed target $X = \bar{X}$, the national authority need not specify which region must remove which units. The relevant conditional cost-minimization problem is

$$\min_{\{x_j\}} \sum_{j=1}^J [C_j(x_j; \xi_j) - S_j(x_j) + D_j(x_j)] \quad (7.3)$$

subject to equation (7.2), $0 \leq x_j \leq V_j$. For unconstrained active suppliers, the Lagrange conditions are

$$\frac{A_j(x_j; \xi_j)}{\kappa_j \omega_j} = q \quad \text{for all unconstrained active } j. \quad (7.4)$$

Equation (7.4) states that unconstrained active regions have the same marginal net cost per effective credit. Regions at $x_j = 0$ or $x_j = V_j$ instead satisfy the corresponding Kuhn–Tucker inequalities. A competitive market reaches the complete Kuhn–Tucker system without the national government assigning region-specific quantities. A common removal subsidy equal to q per effective credit also induces the same supply responses, but it fixes the price rather than the aggregate quantity unless it is adjusted to hit $\bar{X}$.

7.3 Uncertainty and the Price-Quantity Comparison

Finally, suppose the aggregate supply schedule is uncertain when the instrument is selected. Let $C(X; \varepsilon)$ be minimized aggregate net cost after the regional allocation has been optimized as above. Let $B(X)$ be the reduced-form national benefit from contraction, including the reduction in H and the optimally adjusted development margin. Around the expected optimum, write

$$B(X) = aX - \frac{b}{2}X^2, \quad b > 0, \quad (7.5)$$

$$C(X; \varepsilon) = \frac{c}{2}X^2 + \varepsilon X, \quad c > 0, \quad (7.6)$$

where $\mathbb{E}\varepsilon = 0$ and $\text{Var}(\varepsilon) = \sigma_\varepsilon^2$. The expected-cost optimum is $X_0 = a/(b + c)$. A quantity policy fixes $X_Q = X_0$. A price policy fixes the common net incentive at $p_0 = cX_0$, producing

$$X_P = X_0 - \varepsilon/c.$$

Proposition 7.1 (Competitive Implementation and Instrument Choice). *The national spatial-credit system has the following properties.*

- (i) *Under complete information, correct verification, price taking, and a land bank that internalizes local benefits, the target $\bar{Q} = Q^*$ implements the full-information first-best, with equilibrium price $q^* = H'(Q^*)$.*
- (ii) *Under complete information and certainty, the credit market and the matched development charge-removal subsidy pair are allocatively equivalent. The price-policy government balance is $q^*\bar{Q}$.*
- (iii) *For any given aggregate contraction target, competitive trading minimizes aggregate net removal cost and weakly dominates any feasible nontradable regional allocation that fails to satisfy the relevant Kuhn–Tucker conditions, including marginal-net-cost equalization among unconstrained active suppliers.*
- (iv) *In the local quadratic uncertainty model,*

$$\mathbb{E}[W_P - W_Q] = \frac{\sigma_\varepsilon^2}{2c^2}(c - b). \quad (7.7)$$

A price policy is preferred when $c > b$, while a quantity policy is preferred when $b > c$.

Proposition 7.1 does not attribute cost information or marginal-cost equalization uniquely to a market. Both a competitive credit price and an appropriately chosen common policy price decentralize heterogeneous regional responses. The market’s distinctive institutional feature is that a cap commits to the aggregate quantity while trading allocates that quantity cost effectively. Whether quantity commitment is desirable depends on part (iv), not on a general presumption in favor of markets.

Appendix A.7 proves parts (i)–(iii) and states the Kuhn–Tucker conditions for regions at either boundary. Appendix A.8 derives part (iv). Appendix A.8.1 shows, under symmetric interior supply and an equicorrelated shock structure, how pooling multiple regions flattens aggregate marginal cost and reduces aggregate cost uncertainty. The result is presented as a conditional benchmark rather than a general theorem. Appendix A.11 provides a small numerical illustration comparing a tradable aggregate target with a nontradable equal regional allocation; it does not compare trading with an optimally calibrated common price policy.

8 Verification and Target Miss

The preceding market analysis assumed perfect measurement: one certified credit represented exactly the amount of effective physical contraction recorded by the model. We now relax that assumption. Distinguish the certified coefficient $\widehat{\omega}_j$ from the true coefficient ω_j . The registry uses $\widehat{\omega}_j$ when issuing credits and therefore clears the book market according to

$$\eta y + R^G = A_0 + \sum_{j=1}^J \kappa_j \widehat{\omega}_j x_j, \quad (8.1)$$

so that the recorded net floor stock is

$$\eta y - \sum_{j=1}^J \kappa_j \widehat{\omega}_j x_j = A_0 - R^G = \bar{Q}. \quad (8.2)$$

The physical stock, however, depends on the true additionality and permanence of the removal. Physical realization is therefore

$$Q^{\text{real}} = \eta y - \sum_{j=1}^J \kappa_j \omega_j x_j. \quad (8.3)$$

Lemma 8.1 (Verification Error and Target Miss). *The difference between the realized national net floor stock and the book target is*

$$Q^{\text{real}} - \bar{Q} = \sum_{j=1}^J \kappa_j (\widehat{\omega}_j - \omega_j) x_j. \quad (8.4)$$

If certification weakly overstates effective contraction in every region and strictly overstates it in at least one active region, then $Q^{\text{real}} > \bar{Q}$ even though the book market clears exactly.

Subtracting the book identity from the physical identity proves the lemma. Lemma 8.1 is therefore an accounting identity with direct institutional content. A permit registry can be internally balanced while the physical quantity target is missed. Over-certification can arise because removal would have occurred without the program, because rebuilding restrictions are not permanent, or because the same contraction is counted more than once. Additionality is intrinsically counterfactual, while permanence requires ongoing monitoring; neither is resolved by market clearing.

The lemma deliberately does not state a general welfare-loss formula. At a correctly chosen interior allocation, the planner's first-order conditions imply that a smooth, conditionally efficient

perturbation around the optimum has no first-order welfare effect. This observation does not characterize the full welfare effect of an equilibrium generated by incorrect certification coefficients. Appendix A.9 states this limited qualification. The central result here is the quantity miss itself.

9 Institutional Implications

The model separates four functions that are often assigned to a single policy instrument. Table 2 summarizes the mapping.

Table 2: Distinct Failures and Institutional Responses

Problem	Model object	Institutional response
National quantity externality	$H(Q)$ and the target $\bar{Q}$	National authority sets baseline allowances and retirement
Interregional allocation	Heterogeneous $A_j/(\kappa_j\omega_j)$	Competitive trading reallocates the target toward low-cost suppliers
Local political under-removal	$g_j(\ell_j^m) < \bar{g}_j$ or the generalized wedge $\Delta_j > 0$	Land bank or independent local restructuring body internalizes aggregate neighborhood benefits
Verification failure	$\hat{\omega}_j \neq \omega_j$	Additionality tests, rebuilding restrictions, registry controls, audits, and sanctions

9.1 Why a Land Bank Remains Necessary

A competitive market cannot correct a region's refusal to supply credits when the local decision is made by a decisive voter who underweights aggregate damage. Under equal incidence, equation (5.18) is unchanged by the credit price. Credit revenue can increase both the majority quantity and the locally efficient supply quantity, but it shifts their marginal conditions in parallel. Under heterogeneous incidence, revenue can change the wedge, yet its direction depends on who receives it. A land bank is therefore not redundant: it is the institution that replaces the decisive resident's exposure with the population-weighted aggregate exposure when choosing removal.

The corner case $x_j^M = 0 < x_j^T$ is especially important. It describes a region in which the majority rejects every positive removal proposal even though local aggregate welfare favors removal. Appendix A.2 gives explicit conditions for this corner, while Appendix A.6 shows how the result

changes under proportional removal, high-damage-first removal, fringe-first removal, and a fully location-specific removal vector.

9.2 Certification as a Quantity Institution

Verification is not merely a condition for fair payment. It is part of the quantity-control mechanism. If $\widehat{\omega}_j > \omega_j$, the market allows too much development relative to physical contraction. A coherent system therefore requires ex ante eligibility rules, a registry linking each credit to a specific property and legal restriction, monitoring of rebuilding, procedures for reversal or replacement, and controls against double issuance.

9.3 Supporting Urban Policy and TIF

The baseline permits the growing city to choose development subject to the credit price and local costs in G . Public investment can raise the value of development, but its incidence and fiscal cost require separate analysis. Appendix A.10 introduces TIF with a capitalization parameter $\vartheta \in [0, 1]$ and shows that TIF raises private credit demand only to the extent that real public-investment benefits are capitalized into rents, land values, or developer returns. Uncapitalized benefits require a development agreement, public subsidy, or direct public provision. Keeping TIF in the Appendix prevents it from obscuring the paper’s central distinction among politics, allocation, and verification.

9.4 Empirical Implications

The theory points to measurable political wedges. In the equal-incidence benchmark, the object is

$$g_j(\ell_j^m) - \bar{g}_j. \quad (9.1)$$

Estimating it requires geocoded vacancy, measures of physical risk and neighborhood damage, the location and type of removal, and the residential population distribution. Under heterogeneous incidence, the relevant object is equation (5.17), which additionally requires information on who pays removal costs, who captures reuse benefits, and who receives credit revenue. The present paper does not calibrate these objects. It provides the theoretical structure needed to interpret future municipal-level measurement.

10 Conclusion

This paper develops a spatial-credit framework for an economy in which population and the durable floor-space stock contract at different speeds. The central message is institutional rather than market-centric. A spatial-credit market can implement a national net-floor-stock target cost effectively across heterogeneous regions, but it cannot by itself correct political under-removal within a shrinking region, and it cannot guarantee the physical target when certification is inaccurate.

The local political result is driven by the difference between population-weighted average exposure and exposure at the decisive location. Under equal incidence, majority voting yields too little removal when $g_j(\ell_j^m) < \bar{g}_j$. The decomposition in equation (4.5) shows that the wedge arises from two distinct sources: curvature in spatial exposure and the mean-median structure of residential location. An extension with heterogeneous incidence shows why single-peakedness, the population-weighted median ideal point, and identification with the median-location resident must be proved in that order. It also shows that the neutrality of credit revenue is conditional on equal incidence.

At the national level, the relevant quantity is the physical net floor stock $Q = \eta y - \sum_j \kappa_j \omega_j x_j$. The coefficients η and κ_j are technical conversion factors, while local safety, reuse, and neighborhood benefits remain in D_j and S_j . The national external cost $H(Q)$ is limited to burdens shifted across regions through common fiscal and land-use institutions; local costs are counted elsewhere. Under complete information and certainty, cap-and-trade and a matched charge-subsidy policy are allocatively equivalent. For a given target, trading is cost effective relative to nontradable quotas. Under uncertainty, the choice between quantity and price depends on the relative slopes of marginal benefits and costs.

Finally, the distinction between certified and true effective contraction reveals a separate verification failure. Book-market clearing does not imply physical compliance. The policy package must therefore combine a national cap, competitive trading, a land bank that internalizes local benefits, and a verification regime that establishes additionality and permanence. These components address different failures and should be evaluated separately.

Future research should estimate spatial exposure and incidence wedges using property-level data; embed endogenous migration, rents, and firm location in a general-equilibrium model; study strategic behavior by large regions and developers; and analyze differentiated credit classes when removal and development are not sufficiently substitutable to support a single national quantity.

A Proofs, Robustness, and Extensions

The main text states the economic mechanisms and the assumptions needed to interpret them. This Appendix supplies the complete algebra, treats boundary choices, weakens selected sufficient conditions, and records extensions that are useful for robustness but would interrupt the argument in the text.

A.1 Aggregation and Baseline Concavity

Integrating equation (3.6) over the current population gives

$$\int_0^{L_j} \rho_j(\ell) e_j(\ell, x_j) d\ell = \gamma_j \phi_j(z_j(x_j)) \int_0^{L_j} \rho_j(\ell) g_j(\ell) d\ell, \quad (\text{A.1})$$

which equals equation (3.9) by the definition of $\bar{g}_j$. Because $z'_j(x_j) = -1/\bar{N}_j$,

$$-D'_j(x_j) = \frac{\gamma_j N_j}{\bar{N}_j} \bar{g}_j \phi'_j(z_j(x_j)), \quad (\text{A.2})$$

and equation (3.10) follows. Differentiating once more gives equation (3.12). The objective in equation (3.14) is strictly concave because its second derivative is $-A_{jx} < 0$. Equation (3.17) similarly establishes strict concavity of each resident's utility.

A.2 Proof of Proposition 4.1 and Corner Solutions

Under strict concavity, each resident has a unique ideal point. Equation (3.18) implies increasing differences, so ideal points are nondecreasing in ℓ . The population-weighted median ideal point is therefore the ideal point of the resident at ℓ_j^m . At an interior solution it satisfies equation (3.20).

Define

$$\Gamma_j(x; g) \equiv C_{jx}(x; \xi_j) - S'_j(x) - \frac{\gamma_j N_j}{\bar{N}_j} g \phi'_j(z_j(x)). \quad (\text{A.3})$$

Its derivative with respect to x is

$$\Gamma_{jx} = C_{jxx} - S''_j + \frac{\gamma_j N_j}{\bar{N}_j^2} g \phi''_j > 0, \quad (\text{A.4})$$

and $\partial \Gamma_j / \partial g < 0$. Hence the unique interior root is increasing in g . The social root uses $g = \bar{g}_j$ and

the majority root uses $g = g_j(\ell_j^m)$, proving the interior statements.

For corners, write the unique maximizer as the projection of the unconstrained root onto $[0, V_j]$. Monotonicity in g yields the weak inequalities. In particular,

$$x_j^M = 0 < x_j^T \quad (\text{A.5})$$

whenever

$$S'_j(0) - C_{jx}(0; \xi_j) + \frac{\gamma_j N_j}{\bar{N}_j} g_j(\ell_j^m) \phi'_j(z_j(0)) \leq 0 \quad (\text{A.6})$$

but

$$S'_j(0) - C_{jx}(0; \xi_j) + \frac{\gamma_j N_j}{\bar{N}_j} \bar{g}_j \phi'_j(z_j(0)) > 0. \quad (\text{A.7})$$

A.3 Proof of Proposition 4.2 and the Density Corollaries

Equation (4.5) is an identity. Under convex g_j , Jensen's inequality gives

$$\bar{g}_j = \frac{1}{N_j} \int_0^{L_j} \rho_j(\ell) g_j(\ell) d\ell \geq g_j(\bar{\ell}_j). \quad (\text{A.8})$$

Because $g'_j > 0$ and $\ell_j^m \leq \bar{\ell}_j$,

$$g_j(\bar{\ell}_j) \geq g_j(\ell_j^m). \quad (\text{A.9})$$

Strictness follows under the conditions in Proposition 4.2. Proposition 4.1 then gives the quantity comparison.

For constant density,

$$\ell_j^m = \bar{\ell}_j = L_j/2, \quad (\text{A.10})$$

and linear exposure gives $g_j = 1/2$ at both the median and mean.

For exponential density, let $t = \lambda_{dj} L_j$. Direct integration gives

$$\ell_j^m = -\frac{1}{\lambda_{dj}} \log\left(\frac{1 + e^{-t}}{2}\right), \quad \bar{\ell}_j = \frac{1}{\lambda_{dj}} - \frac{L_j e^{-t}}{1 - e^{-t}}. \quad (\text{A.11})$$

The inequality $\ell_j^m < \bar{\ell}_j$ is equivalent to

$$f(t) \equiv 1 - \frac{t}{e^t - 1} + \log\left(\frac{1 + e^{-t}}{2}\right) > 0. \quad (\text{A.12})$$

Since $f(0) = 0$ and

$$f'(t) = \frac{e^t}{(e^t - 1)^2} [t - 2 \tanh(t/2)] > 0 \quad (\text{A.13})$$

for $t > 0$, because $\tanh u < u$, the result follows. The asymptotic expressions are immediate.

A.4 Weighted Median Ideal Points and Weaker Single Crossing

Strict concavity implies that $u_j(\ell, \cdot; q)$ is single peaked for every ℓ . Let $x_j^{\text{dec}}(q)$ satisfy equation (5.10). For any alternative $x > x_j^{\text{dec}}(q)$, at least half of the population has an ideal point weakly below $x_j^{\text{dec}}(q)$ and therefore weakly prefers $x_j^{\text{dec}}(q)$ to x . For any $x < x_j^{\text{dec}}(q)$, at least half has an ideal point weakly above $x_j^{\text{dec}}(q)$ and weakly prefers $x_j^{\text{dec}}(q)$. Hence $x_j^{\text{dec}}(q)$ is a Condorcet winner. This argument uses the population weights induced by ρ_j and does not require ideal points to be ordered by location.

If $u_{jx\ell} \geq 0$ everywhere, standard monotone-comparative-statics reasoning implies that the unique ideal point is nondecreasing in ℓ . Because ℓ_j^m divides population mass in half, the ideal point of the median-location resident is then a weighted median of the ideal-point distribution. Under uniqueness, it is $x_j^{\text{dec}}(q)$.

For an interior ideal point, $u_{jx}(\ell, x_j^I(\ell; q); q) = 0$. The implicit-function theorem gives equation (5.12). Since $u_{jxx} < 0$, it is enough that

$$u_{jx\ell}(\ell, x_j^I(\ell; q); q) \geq 0 \quad (\text{A.14})$$

along the ideal-point locus. This condition is weaker than increasing differences on the entire policy domain but does not by itself handle changes between interior and corner ideals. The main text therefore uses the global condition as a safe sufficient condition.

A.5 Proof of Proposition 5.2

Differentiating equation (5.1) gives

$$\begin{aligned} u_{jx}(\ell, x_j; q) = & -\tau_j(\ell)C_{jx} + s_j(\ell)S'_j + r_j(\ell)q\kappa_j\omega_j \\ & + \frac{\gamma_j}{N_j}g_j(\ell)\phi'_j. \end{aligned} \quad (\text{A.15})$$

Integrating with the adding-up conditions yields equation (5.16). Subtracting N_j times the decisive resident's marginal utility gives equation (5.17). Under equation (5.5), the first three incidence

terms vanish, proving equation (5.18). In that case $r'_j(\ell) = 0$, the credit price does not alter the spatial ordering of ideal points, and neutrality holds globally. Under heterogeneous incidence, differentiating the remaining credit-incidence term with respect to q gives equation (5.19) only on a local price interval over which the decisive voter and the relevant ordering remain unchanged. The qualification follows because q enters equation (5.11) whenever $r'_j(\ell) \neq 0$.

A.6 Targeted and Location-Specific Removal

The proportional benchmark in equation (3.3) treats removal as reducing a common vacancy rate while leaving the spatial exposure profile $g_j(\ell)$ fixed. This is a useful reduced form but does not specify which properties are removed. Three alternatives clarify robustness.

First, suppose removal is targeted toward the highest-damage properties. Let the effective reduction in the vacancy index generated by targeted removal be $h_j(x_j)$, with $h_j(0) = 0$, $h'_j(x_j) > 0$, $h''_j(x_j) \leq 0$, and $0 \leq h_j(x_j) \leq z_j(0)$. Thus $h_j(x_j)$ is measured in the same units as the vacancy index, rather than in units of utility loss or damage, and targeted removal may generate different effective reductions in that index for the same nominal amount removed. Write

$$e_j(\ell, x_j) = \gamma_j g_j(\ell) \phi_j(z_j(0) - h_j(x_j)). \quad (\text{A.16})$$

The political comparison still depends on $\bar{g}_j - g_j(\ell_j^m)$ when the targeting function h_j is common across residents. Targeting changes the magnitude of marginal benefits but not the exposure ordering.

Second, suppose removal begins at the fringe. Let $b_j(x_j)$ be the inward boundary of the remaining vacant zone. Exposure becomes $g_j(\ell; b_j(x_j))$. The political wedge is then

$$\frac{1}{N_j} \int_0^{L_j} \rho_j(\ell) \left[-\frac{\partial e_j(\ell, x_j)}{\partial x_j} \right] d\ell - \left[-\frac{\partial e_j(\ell_j^m, x_j)}{\partial x_j} \right]. \quad (\text{A.17})$$

The baseline result survives if marginal exposure reduction remains larger on average than at the median location. The fixed- g_j model is therefore a local approximation to a broader marginal-exposure comparison.

Third, let $n_j^V(v) \geq 0$ denote the pre-policy density of vacant floor space at property location v , and let $x_j(v) \in [0, n_j^V(v)]$ denote the density of removal at that location. Let $K_j(\ell, v)$ denote the damage kernel from a vacancy at location v to a resident at location ℓ . The integral below is taken over the relevant set of property locations in region j , such as its inherited urban footprint. Resident

damage is

$$E_j(\ell, \mathbf{x}_j) = \int K_j(\ell, v) \left[n_j^V(v) - x_j(v) \right] dv. \quad (\text{A.18})$$

The local planner values removal at v according to the population-weighted average kernel

$$\bar{K}_j(v) = \frac{1}{N_j} \int_0^{L_j} \rho_j(\ell) K_j(\ell, v) d\ell, \quad (\text{A.19})$$

while a decisive resident values it according to $K_j(\ell_j^d, v)$. Political under-removal at location v arises when $K_j(\ell_j^d, v) < \bar{K}_j(v)$. Thus the baseline scalar wedge is the one-dimensional counterpart of a location-specific kernel wedge.

A.7 Planner and Market Implementation

Differentiating equation (6.7) gives

$$\frac{\partial W}{\partial x_j} = -A_j(x_j; \xi_j) + \kappa_j \omega_j H'(Q), \quad (\text{A.20})$$

$$\frac{\partial W}{\partial y} = F_{0y}(y; m) - \eta H'(Q), \quad (\text{A.21})$$

which yield equations (6.8) and (6.9). Under the maintained concavity conditions, the term $-H(Q)$ contributes a negative-semidefinite Hessian. Hence the planner's objective is concave; strict concavity of regional net benefits and F_0 ensures uniqueness over the relevant domain.

Set $\bar{Q} = Q^*$ in equation (6.11). Regional supply and developer demand satisfy equations (6.14) and (6.16). With $q = q^* = H'(Q^*)$, these conditions coincide with the planner conditions, and accounting gives $Q = Q^*$. This proves part (i) of Proposition 7.1. Part (ii) follows because the matched charge and subsidy reproduce the same private first-order conditions; equation (7.1) is an accounting identity.

For part (iii), form the Lagrangian

$$\mathcal{L} = \sum_j [C_j - S_j + D_j] + q \left(\bar{X} - \sum_j \kappa_j \omega_j x_j \right). \quad (\text{A.22})$$

Using equations (3.10) and (3.11), the marginal net removal cost entering the corner conditions is

$$A_j(x_j; \xi_j) = C_{jx}(x_j; \xi_j) - S'_j(x_j) - \frac{\gamma_j N_j}{\bar{N}_j} \bar{g}_j \phi'_j(z_j(x_j)). \quad (\text{A.23})$$

The denominator is $\bar{N}_j$, consistent with equations (3.10), (3.16), and the aggregation formulas above. For any region with $\kappa_j \omega_j > 0$, the Kuhn–Tucker conditions are

$$\frac{A_j(x_j; \xi_j)}{\kappa_j \omega_j} \begin{cases} \geq q, & x_j = 0, \\ = q, & 0 < x_j < V_j, \\ \leq q, & x_j = V_j. \end{cases} \quad (\text{A.24})$$

Thus equation (7.4) is the equality condition only for unconstrained active suppliers; it is not imposed on regions at either boundary. Convexity of net regional cost makes the full Kuhn–Tucker system sufficient. Any feasible nontradable allocation that violates this system admits a cost-reducing feasible reallocation, whereas an allocation satisfying it is conditionally cost minimizing.

A.8 Prices versus Quantities and Regional Pooling

Under the quantity policy, welfare is

$$W_Q = B(X_0) - C(X_0; \varepsilon). \quad (\text{A.25})$$

Under the price policy, the realized condition is $cX_P + \varepsilon = cX_0$, so $X_P = X_0 - \varepsilon/c$. Substitution into equations (7.5) and (7.6), expansion around X_0 , and use of $(a - bX_0) - cX_0 = 0$ give

$$W_P - W_Q = \frac{c - b}{2c^2} \varepsilon^2. \quad (\text{A.26})$$

Taking expectations proves equation (7.7).

Structural interpretation of b . In the one-development-margin benchmark, at a point where $H''(Q) > 0$, define the value of an aggregate effective-contraction quantity X by

$$B(X) \equiv \max_y \{F_0(y; m) - H(\eta y - X)\}. \quad (\text{A.27})$$

Writing $Q = \eta y - X$, an interior solution satisfies

$$F_{0y}(y; m) = \eta H'(Q). \quad (\text{A.28})$$

The envelope theorem gives $B'(X) = H'(Q)$. Differentiating the interior condition and substituting the induced response of y yields

$$b \equiv -B''(X) = \frac{[-F_{0yy}(y; m)]H''(Q)}{[-F_{0yy}(y; m)] + \eta^2 H''(Q)} > 0. \quad (\text{A.29})$$

Thus the marginal-benefit schedule is steeper when deviations in the national net floor stock generate rapidly increasing cross-regional costs and when the development margin is relatively inflexible. These conditions increase the relative attraction of a quantity instrument. This expression is a structural interpretation for the one-development-margin benchmark, not an unconditional identity for a general multivariate development model.

A.8.1 Conditional pooling benchmark

Normalize effective credit coefficients to one and suppose all regions are interior with

$$A_j(x_j; \varepsilon_j) = a + cx_j + \varepsilon_j, \quad c > 0. \quad (\text{A.30})$$

For a given total $X = \sum_j x_j$, equalization implies

$$a + cx_j + \varepsilon_j = \mu \quad (\text{A.31})$$

for all j . Summing gives

$$\mu = a + \bar{\varepsilon} + \frac{c}{J}X, \quad \bar{\varepsilon} = \frac{1}{J} \sum_j \varepsilon_j. \quad (\text{A.32})$$

If shocks have common variance σ^2 and pairwise correlation ϱ ,

$$\text{Var}(\bar{\varepsilon}) = \sigma^2 \left[\varrho + \frac{1 - \varrho}{J} \right]. \quad (\text{A.33})$$

Pooling therefore flattens aggregate marginal cost and, for $\varrho < 1$, lowers aggregate uncertainty. The formula requires common slopes, interior supply, normalized coefficients, and equicorrelation. With heterogeneous slopes or corners, aggregate marginal cost is generally piecewise.

A.9 Verification Error and the Scope of the Quantity Identity

Subtracting equation (8.2) from equation (8.3) proves equation (8.4). Lemma 8.1 is a quantity identity and does not require a welfare approximation. At a correctly chosen interior allocation, the planner's first-order conditions imply that a smooth, conditionally efficient perturbation around the optimum has no first-order welfare effect. This observation does not characterize the full welfare effect of an equilibrium generated by incorrect certification coefficients.

For completeness, one may define the indirect welfare function conditional on a physical value of the national net floor stock as

$$\tilde{W}(Q) \equiv \max_{\mathbf{x}, y} \left\{ W(\mathbf{x}, y) : \eta y - \sum_j \kappa_j \omega_j x_j = Q \right\}. \quad (\text{A.34})$$

Under the maintained concavity conditions, $\tilde{W}''(Q^*) \leq 0$ whenever the second derivative exists. Let E denote a sufficiently small scalar deviation in the realized physical net floor stock. Only when the remaining variables are conditionally reoptimized for each given physical Q does the local expansion

$$\tilde{W}(Q^* + E) - \tilde{W}(Q^*) = \frac{1}{2} \tilde{W}''(Q^*) E^2 + o(E^2) \quad (\text{A.35})$$

follow at an interior optimum. It is not a statement that the welfare loss in a general equilibrium generated by incorrect certification coefficients is necessarily second order.

A.10 TIF Extension

Let $\alpha \in [0, \bar{\alpha}]$ be TIF intensity, $R(y, \alpha)$ the real benefit of public contributions, and $\Psi(\alpha)$ their fiscal and interjurisdictional cost. Let $\vartheta \in [0, 1]$ be the share of public-investment benefit capitalized into rents, land values, or developer returns. The developer's private objective is

$$F^P(y; m, \alpha) = F_0(y; m) + \vartheta R(y, \alpha) - q\eta y. \quad (\text{A.36})$$

The social contribution of TIF is $R(y, \alpha) - \Psi(\alpha)$. An interior social choice satisfies

$$R_\alpha(y, \alpha) = \Psi'(\alpha). \quad (\text{A.37})$$

The developer's marginal demand condition is

$$F_{0y}(y; m) + \vartheta R_y(y, \alpha) = q\eta. \quad (\text{A.38})$$

Under the local concavity condition

$$F_{0yy}(y; m) + \vartheta R_{yy}(y, \alpha) < 0,$$

differentiating the developer's first-order condition shows that TIF raises private credit demand locally if and only if $\vartheta R_{y\alpha}(y, \alpha) > 0$. When $\vartheta < 1$, part of the social benefit is not internalized by developers. A development agreement, public subsidy, or direct public investment is then required to implement the social development margin. This extension preserves the distinction between real public improvements and fiscal relabeling emphasized by [Dye and Merriman \(2000\)](#).

A.11 Numerical Illustration of Conditional Cost Efficiency

Consider three regions with effective coefficients normalized to one and marginal net removal costs

$$A_1(x_1) = 1.5 + 0.03x_1, \quad A_2(x_2) = 2.5 + 0.05x_2, \quad A_3(x_3) = 4.0 + 0.08x_3. \quad (\text{A.39})$$

For the target $X = 90$, cost minimization equalizes active marginal costs. The solution is

$$q = 3.5625, \quad (x_1, x_2, x_3) = (68.75, 21.25, 0). \quad (\text{A.40})$$

The minimized net cost is

$$\int_0^{68.75} A_1(s)ds + \int_0^{21.25} A_2(s)ds = 238.44. \quad (\text{A.41})$$

A nontradable equal quota (30, 30, 30) has net cost

$$\sum_{j=1}^3 \int_0^{30} A_j(s)ds = 312.00. \quad (\text{A.42})$$

Trading therefore lowers cost by 23.6 percent relative to this nontradable quota. The comparison is not between trading and an optimally calibrated common price policy: under certainty, the latter would reproduce the trading allocation. The example illustrates only conditional cost efficiency for

a fixed aggregate target.

References

- Alonso, William. 1964. *Location and Land Use: Toward a General Theory of Land Rent*. Cambridge, MA: Harvard University Press.
- Alvayay Torrejón, Camila, Dusan Paredes, and Mark Skidmore. 2023. “Impact of Demolitions on Neighboring Property Values in Detroit.” *Journal of Regional Science* 63(5): 1073–1099. doi:10.1111/jors.12654.
- Black, Duncan. 1948. “On the Rationale of Group Decision-making.” *Journal of Political Economy* 56(1): 23–34. doi:10.1086/256633.
- Brueckner, Jan K., and Robert W. Helsley. 2011. “Sprawl and Blight.” *Journal of Urban Economics* 69(2): 205–213. doi:10.1016/j.jue.2010.09.003.
- Carpenter, Bruce E., and Dennis R. Heffley. 1982. “Spatial-equilibrium Analysis of Transferable Development Rights.” *Journal of Urban Economics* 12(2): 238–261. doi:10.1016/0094-1190(82)90017-1.
- Coase, Ronald H. 1960. “The Problem of Social Cost.” *Journal of Law and Economics* 3: 1–44.
- Dye, Richard F., and David F. Merriman. 2000. “The Effects of Tax Increment Financing on Economic Development.” *Journal of Urban Economics* 47(2): 306–328. doi:10.1006/juec.1999.2149.
- Fujita, Masahisa. 1989. *Urban Economic Theory: Land Use and City Size*. Cambridge: Cambridge University Press.
- Glaeser, Edward L., and Joseph Gyourko. 2005. “Urban Decline and Durable Housing.” *Journal of Political Economy* 113(2): 345–375. doi:10.1086/427465.
- Levinson, Arik. 1997. “Why Oppose TDRs? Transferable Development Rights Can Increase Overall Development.” *Regional Science and Urban Economics* 27(3): 283–296. doi:10.1016/S0166-0462(96)02163-1.
- McConnell, Virginia, Margaret Walls, and Elizabeth Kopits. 2006. “Zoning, TDRs and the Density of Development.” *Journal of Urban Economics* 59(3): 440–457. doi:10.1016/j.jue.2005.12.005.
- Mills, David E. 1980. “Transferable Development Rights Markets.” *Journal of Urban Economics* 7(1): 63–74. doi:10.1016/0094-1190(80)90026-1.

- Ministry of Land, Infrastructure, Transport and Tourism. 2023. *Act Partially Amending the Act on Special Measures concerning the Promotion of Vacant House Countermeasures and Related Materials*. Effective December 13, 2023. [In Japanese.]
- Montgomery, W. David. 1972. “Markets in Licenses and Efficient Pollution Control Programs.” *Journal of Economic Theory* 5(3): 395–418. doi:10.1016/0022-0531(72)90049-X.
- Muth, Richard F. 1969. *Cities and Housing: The Spatial Pattern of Urban Residential Land Use*. Chicago: University of Chicago Press.
- National Institute of Population and Social Security Research. 2023. *Population Projections for Japan: 2021–2070*. Population Research Series No. 347. Tokyo. [In Japanese.]
- Paredes, Dusan, and Mark Skidmore. 2017. “The Net Benefit of Demolishing Dilapidated Housing: The Case of Detroit.” *Regional Science and Urban Economics* 66: 16–27. doi:10.1016/j.regsciurbeco.2017.05.009.
- Rossi-Hansberg, Esteban, Pierre-Daniel Sarte, and Raymond Owens III. 2010. “Housing Externalities.” *Journal of Political Economy* 118(3): 485–535. doi:10.1086/653138.
- Sadayuki, Taisuke, Yuki Kanayama, and Toshi H. Arimura. 2019. “Evaluating the Externality of Vacant Houses in Japan: The Case of Toshima Municipality, Tokyo.” RIEEM Discussion Paper Series No. 1901, Waseda University.
- Stacy, Christina Plerhoples. 2018. “The Effect of Vacant Building Demolitions on Crime under Depopulation.” *Journal of Regional Science* 58(1): 100–115. doi:10.1111/jors.12350.
- Statistics Bureau of Japan. 2024. *Summary Results of the 2023 Housing and Land Survey: Basic Tabulation on Housing and Households*. Tokyo. [In Japanese.]
- Thorsnes, Paul, and Gerald P. W. Simons. 1999. “Letting the Market Preserve Land: The Case for a Market-driven Transfer of Development Rights Program.” *Contemporary Economic Policy* 17(2): 256–266. doi:10.1111/j.1465-7287.1999.tb00680.x.
- vom Hofe, Rainer, Olivier Parent, and Meghan Grabill. 2019. “What to Do with Vacant and Abandoned Residential Structures? The Effects of Teardowns and Rehabilitations on Nearby Properties.” *Journal of Regional Science* 59(2): 228–249. doi:10.1111/jors.12413.
- Weitzman, Martin L. 1974. “Prices vs. Quantities.” *Review of Economic Studies* 41(4): 477–491. doi:10.2307/2296698.