

SBI-FERI Working Paper Series

SBI-FERI WP E-3

Fiscal Backing and Conditional Exchange-Rate Valuation: Currency Mismatch and a Valuation Fold

Kazumasa Oguro

Hosei University

October 9, 2026



The views expressed in this paper are those of the authors and do not necessarily reflect the official position of their affiliated organizations or the SBI Financial and Economic Research Institute.

Fiscal Backing and Conditional Exchange-Rate Valuation: Currency Mismatch and a Valuation Fold

Kazumasa Oguro

Faculty of Economics, Hosei University
Tokyo, Japan

Abstract

This paper studies how fiscal backing and public foreign-currency exposure determine the exchange rate on impact. With sticky domestic prices, depreciation lowers the real value of domestic-currency liabilities but raises the home-currency value of foreign-currency liabilities. Under a global constant-elasticity benchmark, a sufficiently negative net foreign-currency position makes the valuation schedule hump-shaped, with two solutions below its peak, one at the peak, and none above it. Under a local price approximation, only solutions inside the maintained interval are admissible. We derive impact elasticities, a generalized proximity ratio, a signed distance to the peak, and tax-claim thresholds. With fixed backing, Argentina's benchmark lies on the decreasing branch; the classification reverses when the exchange-rate elasticity of backing exceeds about 0.20. For Japan, observed foreign-currency reserves exceed reported contingent drains, although the complete public foreign-currency position is not measured. These model-implied balance-sheet illustrations do not constitute a complete theory of exchange-rate determinacy or crisis timing.

Keywords: fiscal theory of the price level; exchange rate; fiscal backing; consolidated public sector; exchange-rate pass-through; currency mismatch; valuation fold.

JEL Classification: E31; E52; E62; F31; F41.

Correspondence: Kazumasa Oguro, Faculty of Economics, Hosei University, Tokyo, Japan. The author is solely responsible for the views expressed in this paper.

1. Introduction

Why can fiscal deterioration weaken a currency when most public debt is denominated in domestic currency? When does foreign-currency debt make that adjustment nonlinear? These questions matter whenever fiscal news changes the value that investors assign to inherited public liabilities.

The first mechanism is familiar. If nominal liabilities rise without an equal increase in expected fiscal backing, their real value must fall. With sticky domestic prices, the exchange rate can move before the full price level adjusts. Depreciation raises import prices and the CPI, which reduces the real value of domestic-currency liabilities.

Foreign-currency liabilities introduce a second effect. Depreciation raises their value in domestic currency one for one. CPI relief, by contrast, rises less than proportionally when effective short-run pass-through is below one. The value available to support nominal liabilities can therefore become hump-shaped in the exchange rate. We refer to this nonlinearity as a valuation fold.

Under the global constant-elasticity benchmark, liabilities below the peak are consistent with two exchange rates. At the peak there is one solution. Above the peak no exchange rate satisfies the benchmark valuation equation. If the price relation is instead treated as a local approximation, only solutions inside its maintained pricing interval are economically admissible.

The multiplicity is a property of the valuation relation, not a complete result on monetary–fiscal determinacy. The same inherited liabilities and fiscal backing can support two static valuation solutions when currency mismatch makes the schedule nonmonotone. Additional restrictions on policy behavior, expectations, and financial-market equilibrium are needed to select between them. Continuity from the zero-mismatch benchmark provides a useful static refinement, but not a dynamic selection rule.

The paper makes four contributions. First, it derives a closed-form map from marginal fiscal backing and effective CPI pass-through to impact depreciation. Second, it shows how currency mismatch and the exchange-rate elasticity of fiscal backing shape the valuation schedule. A negative effective position generates a fold only when the combined elasticity lies between zero and one. Third, it separates national external wealth from public backing. Private foreign assets matter only through the present value of future tax revenue. Fourth, it connects the theory to reproducible balance-sheet illustrations for Argentina and Japan.

We use the two countries for different purposes. Under fixed fiscal backing, Argentina’s benchmark schedule is decreasing at the reference exchange rate. A sensitivity calculation shows that the classification reverses when the exchange-rate elasticity of fiscal backing rises above roughly 0.20. Japan provides a reserve-based partial comparison: observed foreign-currency reserves exceed reported contingent drains, but the complete consolidated public foreign-currency position is not measured. The exercises are model-implied balance-sheet diagnostics, not causal estimates, and they do not place the countries on a common scale.

Throughout the paper, we condition on the inherited balance sheet, expected fiscal backing, and the stated price-setting or financial-market assumption. The model values the public balance sheet at a given date. It does not determine default, peg exit, the subsequent transition path, or equilibrium selection between branches.

The remainder of the paper is organized as follows. Section 2 relates the analysis to fiscal valuation, currency mismatch, pass-through, external valuation, and intervention. Section 3 derives the sectoral and consolidated valuation equations. Section 4 presents the main results. Section 5 provides balance-sheet illustrations for Argentina and Japan. Section 6 concludes. The Appendix contains proofs, extensions, data documentation, and replication details.

2. Related literature

2.1. Fiscal valuation and open-economy policy regimes

The fiscal theory of the price level values nominal public liabilities against expected public resources. Its policy-regime foundations are developed by Sargent and Wallace (1981), Leeper (1991), Sims (1994), and Woodford (1995). Cochrane (2001, 2023) emphasizes the asset-pricing content of the government valuation equation and the role of long-duration debt. Gross issuance is not necessarily unfunded because future surpluses and discount rates may adjust (Bianchi et al., 2023; Jiang et al., 2024). Maturity and market value also shape the amount of inflationary fiscal relief (Hilscher et al., 2022).

Whether the government valuation relation determines the price level remains controversial. McCallum (2001) and Buiter (2002) stress policy rules, equilibrium selection, and the difference between an accounting identity and an equilibrium condition. This paper takes a narrower approach. It conditions on a fiscal-led valuation regime and asks how the price level and exchange rate must adjust at date t when expected public resources do not fully validate inherited nominal liabilities. The multiplicity derived below reinforces the need to distinguish a conditional valuation relation from a complete theory of price-level and exchange-rate determinacy.

Open-economy work shows that fiscal regimes can affect exchange rates. Dupor (2000) and Daniel (2001) study joint price-level and exchange-rate determinacy, while Bergin (2000) examines monetary union. Jiang (2022) studies dynamic fiscal effects on real exchange rates, currency returns, and risk premia. This paper asks a narrower question: how does the consolidated public balance sheet map into the exchange rate at a given date when domestic prices adjust incompletely? The answer yields a sign-based geometry, a valuation bound, and a scale-free measure of the distance to its peak.

2.2. Fiscal currency crises and currency mismatch

Fiscal accounts are central to models of currency crises. Burnside et al. (2001) link prospective fiscal costs from financial guarantees to eventual currency crises. Corsetti and Maćkowiak (2006) study fiscal imbalances and speculative attacks. Those models emphasize expected monetization, reserve exhaustion, regime change, or attack timing. The present model does not contain an attack, a reserve run, or a default decision. It isolates the valuation effect of inherited public liabilities.

A large literature studies the balance-sheet costs of depreciation. Krugman (1999) emphasizes the transfer problem and balance-sheet deterioration. Aghion et al. (2001) study credit constraints, and Céspedes et al. (2004) show how liability dollarization affects firms through net worth. These mechanisms rely on production, collateral, default risk, or financial amplification. The fold derived here needs none of them. It follows from two fiscal effects: CPI adjustment reduces the real value of domestic-currency debt, while depreciation raises the domestic-currency value of foreign-currency debt.

The resulting schedule resembles a debt-Laffer curve (Krugman, 1988), but the economic margin differs. In a conventional debt-Laffer curve, a larger face value can reduce expected repayment by weakening incentives or repayment capacity. Here the exchange rate is the horizontal variable. The peak arises because CPI relief grows less than proportionally, whereas foreign-currency revaluation grows one for one.

Quantitative sovereign-default models endogenize repayment, borrowing costs, and output (Aguiar and Gopinath, 2006; Arellano, 2008). Debt-restructuring studies analyze the contractual adjustment that follows when repayment capacity and market access fail (Sturzenegger and Zettelmeyer, 2006). Original-sin and currency-denomination research separates creditor residence from debt currency (Du and Schreger, 2016; Eichengreen et al., 2003). The valuation bound in this paper is not a default threshold. It marks the point at which the inherited balance sheet cannot be valued under the stated assumptions. Default and restructuring models determine what happens next.

2.3. Nominal rigidity, pass-through, and fiscal real-exchange-rate adjustment

Sticky-price open-economy models provide the background for the short-run price relation. Obstfeld and Rogoff (1995) and Galí and Monacelli (2005) show how exchange-rate movements affect relative prices and consumer prices when domestic prices adjust slowly. Gopinath et al. (2010) document large differences in import-price pass-through across invoicing currencies. Caselli and Roitman (2016) find nonlinear and asymmetric pass-through in emerging markets, especially during large depreciations. Their 10 and 20 percent episode thresholds motivate transparent sensitivity cases; they are not estimates of a universal boundary for the local price relation.

Related work studies the dynamic effects of fiscal shocks on real exchange rates under nominal rigidity (Monacelli and Perotti, 2010; Ravn et al., 2012). This paper instead holds the present value of public resources fixed or specifies its local response. That choice isolates the contemporaneous valuation map and leaves transition dynamics to those richer models.

2.4. Government debt, central-bank balance sheets, and external valuation

Public-sector consolidation is essential. Government bonds held by the central bank are liabilities of the fiscal authority and assets of the central bank. Remittances are transfers within the public sector. Currency and reserve balances held outside the consolidated sector remain public liabilities. Work on central-bank solvency and remittances explains why seigniorage, reserve remuneration, asset income, and fiscal support matter for central-bank resources (Bassetto and Messer, 2013; Del Negro and Sims, 2015; Hall and Reis, 2015; Reis, 2015, 2016). The sectoral equations below make these flows explicit before consolidation.

International-portfolio research measures valuation effects on national wealth. Gourinchas and Rey (2007) identify a financial channel of external adjustment. Lane and Shambaugh (2010) measure the currency composition of external positions, and Bénétrix et al. (2015) document large valuation changes during the global financial crisis. This paper focuses on a narrower object: foreign-currency resources that can support public liabilities. Public foreign assets enter directly. Private foreign assets enter only through the present value of future tax revenue.

Foreign-exchange intervention is usually studied through portfolio balance, signaling, order flow, liquidity, and intermediary risk-bearing capacity (Cavallino, 2019; Evans and Lyons, 2002; Gabaix and Maggiori, 2015; Sarno and Taylor, 2001). The result below concerns only direct consolidated public-sector accounting. It complements rather than replaces those market channels.

Hard pegs require a separate interpretation. A fixed nominal exchange rate removes the exchange rate as an immediate valuation margin, but it does not fix the real exchange rate. The Argentina application therefore treats convertibility as institutional background. It asks what the inherited pre-exit balance sheet implies once the nominal exchange rate is allowed to adjust.

2.5. Research gap

The paper isolates a simple mechanism that the surrounding literatures do not usually present in this form. When CPI-based fiscal relief has a lower exchange-rate elasticity than foreign-currency public-liability revaluation, a negative public foreign-currency position can make the valuation schedule hump-shaped. The closed-form elasticity, valuation bound, distance measure, tax-claim thresholds, and country exercises develop the implications of that mechanism. The paper does not propose a general theory of exchange-rate determination. It provides a date- t valuation model and a transparent way to compare public balance sheets.

3. Model

3.1. Environment, notation, and intraperiod timing

Time is discrete and indexed by $t = 0, 1, 2, \dots$. The home economy is small relative to the rest of the world. Foreign prices and foreign real quantities are therefore taken as given in the date- t valuation exercise.

Let $E_t > 0$ denote the nominal exchange rate, measured as units of home currency per unit of foreign currency. An increase in E_t is a nominal depreciation. Let $P_t > 0$ be the home consumer price index and $P_t^* > 0$ the foreign consumer price index, measured in foreign currency. The level of the real exchange rate is

$$Q_t \equiv \frac{E_t P_t^*}{P_t}. \quad (1)$$

Thus, Q_t is the number of home consumption baskets required to purchase one foreign consumption basket. An increase in Q_t is a real depreciation.

All financial positions carrying a superscript $*$ are nominal amounts denominated and measured in units of foreign currency. If X_{t-1}^* is a foreign-currency claim inherited at the opening of period t , its nominal value in home currency is $E_t X_{t-1}^*$ and its value in home consumption units is $E_t X_{t-1}^* / P_t$. A home-currency nominal claim X_{t-1} has real value X_{t-1} / P_t . Foreign prices P_t^* continue to enter the real exchange rate in eq. (1), but they do not multiply a nominal foreign-currency claim when that claim is converted into home currency.

The public sector consists of a fiscal authority, indexed by G , and a central bank, indexed by C . Financial positions dated $t - 1$ are inherited at the opening of period t and valued at period- t prices. The main text treats nominal public liabilities as one-period instruments. Appendix D explains the conditional extension to long-duration liabilities.

The timing isolates the impact valuation margin. Public liabilities and foreign-currency positions are inherited at the start of the period. The home-goods price and the local-currency component of import prices are fixed for a short interval. Fiscal news then arrives, and the nominal exchange rate can move before the home-goods price is reset. Part of the exchange-rate movement passes into import prices and the CPI.

The resulting change in P_t is a date- t price-level adjustment. It is distinct from expected inflation $\mathbb{E}_t \pi_{t+1} = \mathbb{E}_t (p_{t+1} - p_t)$. The model conditions on the expected future inflation and policy path. It does not determine whether the initial price-level change persists, reverses, or is followed by further inflation.

Let $\mathcal{M}_{t,t+j} > 0$ denote the real stochastic discount factor that values a unit of the home consumption basket delivered at date $t + j$ in date- t home-consumption units, with $\mathcal{M}_{t,t} = 1$. Expectations conditional on date- t information are denoted by $\mathbb{E}_t$. The state-contingent discount factor allows public resources to be valued consistently with real interest rates and risk premia. The subsequent propositions condition on the resulting present value S_t and therefore do not require a separate asset-pricing model for $\mathcal{M}_{t,t+j}$.

3.2. The fiscal authority

Let $B_{t-1}^G > 0$ be the nominal market value of the fiscal authority's home-currency liabilities inherited at the opening of period t . It includes government bonds held by private residents, foreign investors, and the central bank. Let $D_{t-1}^{G*} \geq 0$ denote the fiscal authority's foreign-currency liabilities and $R_{t-1}^{G*} \geq 0$ its foreign-currency financial assets, both denominated and measured in units of foreign currency.

The fiscal authority's opening net liability, measured in home consumption units, is

$$\mathcal{L}_t^G \equiv \frac{B_{t-1}^G}{P_t} + \frac{E_t D_{t-1}^{G*}}{P_t} - \frac{E_t R_{t-1}^{G*}}{P_t}. \quad (2)$$

The first term is the real value of home-currency government liabilities. The second is the home-consumption

value of foreign-currency liabilities. The third enters with a minus sign because foreign assets reduce the fiscal authority's net liability.

Let s_{t+j}^G denote the fiscal authority's real primary surplus in period $t + j$, excluding interest payments and transfers received from the central bank. Let z_{t+j} denote the real remittance from the central bank to the fiscal authority. Define

$$S_t^G \equiv \mathbb{E}_t \sum_{j=0}^{\infty} \mathcal{M}_{t,t+j} s_{t+j}^G, \quad (3)$$

$$Z_t \equiv \mathbb{E}_t \sum_{j=0}^{\infty} \mathcal{M}_{t,t+j} z_{t+j}. \quad (4)$$

The intertemporal budget constraint and the no-Ponzi condition in Appendix A imply the fiscal authority's valuation equation

$$\frac{B_{t-1}^G}{P_t} + \frac{E_t D_{t-1}^{G*}}{P_t} = S_t^G + Z_t + \frac{E_t R_{t-1}^{G*}}{P_t}. \quad (5)$$

3.3. The central bank

Let $B_{t-1}^C \geq 0$ be the nominal market value of government bonds held by the central bank. Let $H_{t-1} > 0$ be the nominal value of central-bank liabilities held outside the central bank, including currency and reserve balances. Let $D_{t-1}^{C*} \geq 0$ and $R_{t-1}^{C*} \geq 0$ denote the central bank's foreign-currency liabilities and foreign reserve assets, denominated and measured in units of foreign currency.

The central bank's opening net liability in home consumption units is

$$\mathcal{L}_t^C \equiv \frac{H_{t-1}}{P_t} + \frac{E_t D_{t-1}^{C*}}{P_t} - \frac{B_{t-1}^C}{P_t} - \frac{E_t R_{t-1}^{C*}}{P_t}. \quad (6)$$

Currency and reserve balances are liabilities. Government bonds and foreign reserves are assets and therefore reduce the central bank's net liability.

Let s_{t+j}^C denote the central bank's real net resource income before remittances. It includes future seigniorage and operating income, less interest paid on reserve balances and operating costs. Returns already capitalized in the inherited market values of assets are not counted a second time. Define

$$S_t^C \equiv \mathbb{E}_t \sum_{j=0}^{\infty} \mathcal{M}_{t,t+j} s_{t+j}^C. \quad (7)$$

The remittance z_{t+j} is an outflow from the central bank. The intertemporal budget constraint and the no-Ponzi condition in Appendix A imply

$$\frac{H_{t-1}}{P_t} + \frac{E_t D_{t-1}^{C*}}{P_t} + Z_t = \frac{B_{t-1}^C}{P_t} + \frac{E_t R_{t-1}^{C*}}{P_t} + S_t^C. \quad (8)$$

3.4. Consolidation

Adding eqs. (5) and (8) eliminates Z_t , because a remittance is revenue for the fiscal authority and an equal expenditure for the central bank. Government bonds held by the central bank also cancel because they are a fiscal-authority liability and a central-bank asset.

Define consolidated home-currency nominal public liabilities by

$$\mathcal{B}_{t-1} \equiv B_{t-1}^G - B_{t-1}^C + H_{t-1}. \quad (9)$$

Thus, $\mathcal{B}_{t-1}$ is government debt held outside the central bank plus currency and reserve balances. Define consolidated foreign-currency liabilities, foreign assets, and future public-sector resources by

$$D_{t-1}^* \equiv D_{t-1}^{G*} + D_{t-1}^{C*}, \quad (10)$$

$$R_{t-1}^* \equiv R_{t-1}^{G*} + R_{t-1}^{C*}, \quad (11)$$

$$S_t \equiv S_t^G + S_t^C. \quad (12)$$

The consolidated public-sector valuation equation is

$$\frac{\mathcal{B}_{t-1}}{P_t} + \frac{E_t D_{t-1}^*}{P_t} = S_t + \frac{E_t R_{t-1}^*}{P_t}. \quad (13)$$

The consolidated equation treats fiscal and central-bank resources as fungible in present value. Actual transfers may face legal or political constraints. In that case, the sectoral location of assets and liabilities can matter for implementation even though the consolidated equation is unchanged.

3.5. Private foreign assets and the public tax claim

Let $A_{t-1}^* \geq 0$ denote private foreign assets denominated and measured in units of foreign currency. They are not owned by the public sector and cannot be added directly to R_{t-1}^* . Let τ_{t+j}^A be real tax revenue in period $t+j$ generated by income, realized gains, or transfers associated with the private foreign-asset base. Its present value is

$$\mathcal{T}_t^A \equiv \mathbb{E}_t \sum_{j=0}^{\infty} \mathcal{M}_{t,t+j} \tau_{t+j}^A. \quad (14)$$

For a parsimonious local representation, suppose

$$\mathcal{T}_t^A = \theta \frac{E_t A_{t-1}^*}{P_t}, \quad 0 \leq \theta \leq 1. \quad (15)$$

The coefficient θ is the present-value tax-claim coefficient. It measures the share of the current home-consumption value of private foreign assets represented by credible future tax revenue. It is neither a confiscation rate nor a one-period statutory tax rate. To avoid double counting, this revenue is excluded from S_t .

Using eq. (15), the consolidated valuation equation becomes

$$\frac{\mathcal{B}_{t-1}}{P_t} = S_t + \frac{E_t}{P_t} \mathcal{F}_{t-1}^*, \quad (16)$$

where the public sector's net foreign-currency fiscal position is

$$\mathcal{F}_{t-1}^* \equiv R_{t-1}^* - D_{t-1}^* + \theta A_{t-1}^*. \quad (17)$$

Multiplying eq. (16) by P_t yields the nominal form

$$\mathcal{B}_{t-1} = P_t S_t + E_t \mathcal{F}_{t-1}^*. \quad (18)$$

3.6. Consumer prices and incomplete pass-through

The home household consumes a Cobb–Douglas aggregate of home goods and imported goods,

$$C_t = \left(\frac{C_{H,t}}{1-\alpha} \right)^{1-\alpha} \left(\frac{C_{M,t}}{\alpha} \right)^{\alpha}, \quad 0 < \alpha < 1, \quad (19)$$

where α is the expenditure share of imports. Let $P_{H,t}$ denote the home-currency price of the home good and $P_{M,t}$ the home-currency price of the imported good. Expenditure minimization gives

$$P_t = P_{H,t}^{1-\alpha} P_{M,t}^{\alpha}. \quad (20)$$

During the impact pricing interval, the home-goods price $P_{H,t}$ is predetermined. Import prices satisfy the reduced-form pass-through schedule

$$P_{M,t} = \Psi_t (E_t P_t^*)^{\lambda}, \quad 0 \leq \lambda \leq 1, \quad (21)$$

where $\Psi_t > 0$ is a predetermined local-currency pricing component and λ is the short-run pass-through of the exchange rate and foreign price into the import price. The import share α and pass-through λ are conceptually distinct. Define effective short-run CPI pass-through by

$$\eta \equiv \alpha\lambda, \quad 0 \leq \eta < 1. \quad (22)$$

Substituting eq. (21) into eq. (20) gives

$$P_t = \Omega_t E_t^{\eta}, \quad (23)$$

where

$$\Omega_t \equiv P_{H,t}^{1-\alpha} \Psi_t^{\alpha} (P_t^*)^{\eta} > 0 \quad (24)$$

collects all components predetermined within the pricing interval. Combining eqs. (18) and (23) gives the sticky-price valuation equation

$$\mathcal{B}_{t-1} = S_t \Omega_t E_t^{\eta} + \mathcal{F}_{t-1}^* E_t. \quad (25)$$

3.7. Fiscal backing and local elasticities

The benchmark holds S_t fixed at date t . To allow local feedback from liabilities and the exchange rate, write

$$S_t = S_t(\mathcal{B}_{t-1}, E_t), \quad S_t > 0. \quad (26)$$

Define the partial elasticities

$$\xi_t \equiv \frac{\partial \log S_t}{\partial \log \mathcal{B}_{t-1}}, \quad (27)$$

$$\chi_t \equiv \frac{\partial \log S_t}{\partial \log E_t}. \quad (28)$$

The coefficient ξ_t summarizes the marginal response of expected taxes, spending restraint, and central-bank resources to inherited public liabilities. The coefficient χ_t is a reduced-form elasticity that summarizes feedbacks that a fuller general-equilibrium model would determine. A positive χ_t applies when depreciation increases future tax revenue or public resources. A negative χ_t applies when depreciation lowers real income, erodes the tax base, or reduces central-bank net income.

A simple fiscal reaction rule clarifies the interpretation of ξ_t . Let $s_{t+j}^P \equiv s_{t+j}^G + s_{t+j}^C$ denote public-sector net resources before the private foreign-asset tax claim, and suppose

$$s_{t+j}^P = \bar{s}_{t+j} + \phi_j \frac{\mathcal{B}_{t-1}}{P_t}, \quad j \geq 0. \quad (29)$$

Holding E_t and hence the impact price closure fixed, define $\Phi_t \equiv \mathbb{E}_t \sum_{j \geq 0} \mathcal{M}_{t,t+j} \phi_j$. Then

$$\frac{\partial S_t}{\partial \mathcal{B}_{t-1}} = \frac{\Phi_t}{P_t}, \quad \xi_t = \Phi_t \frac{\mathcal{B}_{t-1}}{P_t S_t}. \quad (30)$$

When the foreign-currency position is zero, $\mathcal{B}_{t-1} = P_t S_t$ and $\xi_t = \Phi_t$. Thus, ξ_t summarizes the discounted response of future taxes, spending, and central-bank resources. Equation (29) isolates that response while holding the exchange rate and the short-run price relation fixed. It is not a complete specification of $S_t(\mathcal{B}_{t-1}, E_t)$. The partial derivatives also hold fixed the discount factor, baseline resources, and fiscal-response coefficients. The political determination of $\{\phi_j\}$ lies outside the model.

Remark 1 (Relation between liability and exchange-rate backing elasticities). *If the response term in eq. (29) is the only variable component of public resources and the baseline resources and response coefficient are exchange-rate invariant, then*

$$S_t = \bar{S}_t + \Phi_t \frac{\mathcal{B}_{t-1}}{\Omega_t E_t^\eta}, \quad (31)$$

which implies $\chi_t = -\eta \xi_t$. More generally, for this decomposition assume $\bar{S}_t(E_t) > 0$ and $\Phi_t(E_t) > 0$ and write

$$S_t = \bar{S}_t(E_t) + \Phi_t(E_t) \frac{\mathcal{B}_{t-1}}{\Omega_t E_t^\eta} \quad (32)$$

and define $\varepsilon_{\bar{S},E} \equiv \partial \log \bar{S}_t(E_t) / \partial \log E_t$ and $\varepsilon_{\Phi,E} \equiv \partial \log \Phi_t(E_t) / \partial \log E_t$. Then

$$\chi_t = (1 - \xi_t) \varepsilon_{\bar{S},E} + \xi_t (\varepsilon_{\Phi,E} - \eta). \quad (33)$$

Accordingly, the benchmark $\chi_t = 0$ is a reduced-form exchange-rate-independent-backing benchmark. When $\xi_t > 0$, it generally requires exchange-rate-sensitive baseline resources or fiscal-response coefficients to offset the mechanical effect of CPI adjustment on the real liability-response term:

$$(1 - \xi_t) \varepsilon_{\bar{S},E} + \xi_t \varepsilon_{\Phi,E} = \eta \xi_t. \quad (34)$$

It is not an implication of eq. (29) taken as a complete fiscal rule. Propositions 1 and 3 therefore treat ξ_t and χ_t as distinct local partial elasticities of a more general public-resource schedule. In the complete special case eq. (31), and away from $\xi_t = 1$, Proposition 1 reduces to $d \log E_t / d \log \mathcal{B}_{t-1} = 1/\eta$. The fiscal rule changes backing levels but not the liability elasticity unless additional exchange-rate-sensitive resource components are present.

3.8. Date- t valuation equilibrium

Definition 1 (Date- t valuation equilibrium). *Given inherited public liabilities $\mathcal{B}_{t-1}$, public resources or a local resource schedule $S_t(\mathcal{B}_{t-1}, E_t)$, the public sector's foreign-currency position $\mathcal{F}_{t-1}^*$, foreign prices, and a stated price or financial-market assumption, a date- t valuation equilibrium is a positive pair (P_t, E_t) that satisfies eq. (18) and that assumption.*

The model therefore values the inherited public balance sheet at a given date. It does not describe the full

transition path. Dynamic consistency under uncovered interest parity and market selection between branches remain outside the analysis.

4. Analysis

4.1. Domestic-currency liabilities under the sticky-price closure

Set $\mathcal{F}_{t-1}^* = 0$. The sticky-price valuation equation is

$$\mathcal{B}_{t-1} = \Omega_t E_t^\eta S_t(\mathcal{B}_{t-1}, E_t). \quad (35)$$

Proposition 1 (Generalized impact depreciation). *Suppose $\mathcal{F}_{t-1}^* = 0$ and a locally regular date- t valuation equilibrium exists. If $\eta + \chi_t \neq 0$, the liability elasticity of the nominal exchange rate is*

$$\frac{d \log E_t}{d \log \mathcal{B}_{t-1}} = \frac{1 - \xi_t}{\eta + \chi_t}. \quad (36)$$

If $\eta + \chi_t > 0$, an increase in liabilities produces impact depreciation when $\xi_t < 1$, no exchange-rate change when $\xi_t = 1$, and appreciation when $\xi_t > 1$.

Proposition 1 gives the impact response when the public FX position is zero. The denominator combines CPI adjustment with the response of public resources to depreciation. A positive χ_t reduces the required depreciation because depreciation itself improves backing. A negative χ_t increases it.

The numerator measures how much of the liability increase remains unfunded. When $\xi_t = 1$, expected public resources rise in the same proportion as liabilities. No price-level or exchange-rate adjustment is then required. This local funding boundary is related to, but distinct from, active–passive classifications in dynamic policy-rule models.

Remark 2 (Loss of local valuation regularity). *As $\eta + \chi_t \rightarrow 0^+$, CPI-based valuation relief is nearly offset by an adverse response of expected public resources to depreciation, and the conditional liability elasticity in eq. (36) becomes large. This is a loss of local valuation regularity, not a statement about dynamic instability or an explosive adjustment process. It also does not imply that the marginal public resources required to support an exogenously fixed exchange-rate target diverge. From eq. (103),*

$$\frac{\partial S_t^{\text{req}}(\bar{E})}{\partial \mathcal{B}_{t-1}} = \frac{1}{\Omega_t \bar{E}^\eta}, \quad (37)$$

which is finite. The loss of regularity concerns the exchange-rate adjustment generated by the endogenous backing schedule. This zero-position loss of local valuation regularity is distinct from the linear decreasing $\gamma = 0$ negative-position case characterized in Proposition 5.

Corollary 1 (Exchange-rate-independent backing). *If $\chi_t = 0$, Proposition 1 reduces to*

$$\frac{d \log E_t}{d \log \mathcal{B}_{t-1}} = \frac{1 - \xi_t}{\eta}. \quad (38)$$

If S_t is conditionally fixed, so that $\xi_t = 0$, the unique positive exchange rate is

$$E_t = \left(\frac{\mathcal{B}_{t-1}}{S_t \Omega_t} \right)^{1/\eta}, \quad \eta > 0. \quad (39)$$

The result applies to the short pricing interval. It says that the exchange rate supplies the immediate valuation margin given the inherited balance sheet and predetermined price components. It does not rank the timing of all asset prices along a full transition path.

Remark 3 (Import-price versus effective CPI pass-through). *Because $\eta = \alpha\lambda$, a small import share and weak import-price pass-through both magnify the exchange-rate change needed to produce a given CPI adjustment. They have the same product in the benchmark elasticity but different structural interpretations and potentially different policy implications. A lower import share reduces CPI exposure through expenditure composition, whereas lower import-price pass-through reflects invoicing practices, markups, competitive conditions, and price-setting behavior. Trade or production structure can affect the former, while invoicing currency and market competition can affect the latter. Full import-price pass-through, $\lambda = 1$, does not generally eliminate the fold because effective CPI pass-through remains $\eta = \alpha < 1$ when the import share is below one. Under the constant-elasticity backing specification, the valuation schedule becomes linear at unit CPI-and-backing elasticity, $\eta + \chi = 1$; $\eta = 1$ should therefore be described as unit effective CPI pass-through rather than merely “full pass-through.”*

Remark 4 (Zero effective pass-through). *If $\eta = 0$, $\chi_t = 0$, and $\mathcal{F}_{t-1}^* = 0$, the sticky-price valuation equation contains no E_t . The exchange rate cannot provide the valuation margin in the zero-foreign-position benchmark. Adjustment must then occur through another nominal price, a revision of expected public resources, or a different closure. Accordingly, the level solution in eq. (39) requires $\eta > 0$.*

4.2. A Nonnegative Effective Public Foreign-Currency Position

Suppose $\mathcal{F}_{t-1}^* \geq 0$. Under conditionally fixed S_t , define the valuation-capacity function

$$V_t(E) \equiv S_t \Omega_t E^\eta + \mathcal{F}_{t-1}^* E. \quad (40)$$

Proposition 2 (Existence, uniqueness, and benchmark comparative statics). *Let $S_t > 0$, $\Omega_t > 0$, $0 < \eta < 1$, and $\mathcal{F}_{t-1}^* \geq 0$ be conditionally fixed. For every $\mathcal{B}_{t-1} > 0$, there is a unique positive exchange rate satisfying*

$$\mathcal{B}_{t-1} = V_t(E_t). \quad (41)$$

The benchmark comparative statics are

$$\frac{\partial E_t}{\partial \mathcal{B}_{t-1}} = \frac{1}{\eta S_t \Omega_t E_t^{\eta-1} + \mathcal{F}_{t-1}^*} > 0, \quad (42)$$

$$\frac{\partial E_t}{\partial S_t} = -\frac{\Omega_t E_t^\eta}{\eta S_t \Omega_t E_t^{\eta-1} + \mathcal{F}_{t-1}^*} < 0, \quad (43)$$

$$\frac{\partial E_t}{\partial \mathcal{F}_{t-1}^*} = -\frac{E_t}{\eta S_t \Omega_t E_t^{\eta-1} + \mathcal{F}_{t-1}^*} < 0. \quad (44)$$

Under exchange-rate-dependent fiscal backing, the sign of the public foreign-currency position alone need not determine global geometry. If $V(E) = AE^\gamma + \mathcal{F}^*E$ with $\mathcal{F}^* > 0$ and $\gamma < 0$, the schedule is U-shaped and can have multiple intersections. The monotonicity result therefore requires a positive combined exponent. Proposition 2 satisfies this condition because fixed backing implies $\gamma = \eta > 0$. Under fixed backing, or more generally when the relevant exponent is positive, the sign of the public FX position provides the first-order geometry classification.

Define the shares of consolidated liabilities backed by public resources and the effective foreign-currency

position as

$$\omega_{S,t} \equiv \frac{P_t S_t}{\mathcal{B}_{t-1}}, \quad (45)$$

$$\omega_{F,t} \equiv \frac{E_t \mathcal{F}_{t-1}^*}{\mathcal{B}_{t-1}}. \quad (46)$$

For $\mathcal{F}_{t-1}^* \geq 0$, both shares are nonnegative and satisfy

$$\omega_{S,t} + \omega_{F,t} = 1. \quad (47)$$

Proposition 3 (General Local Elasticity and the Effective FX-Backing Share). *Suppose $\mathcal{F}_{t-1}^* \geq 0$ and a locally regular valuation equilibrium exists. Holding Ω_t and $\mathcal{F}_{t-1}^*$ fixed, the liability elasticity of the nominal exchange rate is*

$$\frac{d \log E_t}{d \log \mathcal{B}_{t-1}} = \frac{1 - \omega_{S,t} \xi_t}{\omega_{S,t}(\eta + \chi_t) + \omega_{F,t}}. \quad (48)$$

In the benchmark $\chi_t = 0$, define

$$\Lambda_t \equiv \eta \omega_{S,t} + \omega_{F,t}. \quad (49)$$

Then

$$\frac{d \log E_t}{d \log \mathcal{B}_{t-1}} = \frac{1 - \omega_{S,t} \xi_t}{\Lambda_t}, \quad (50)$$

and

$$\frac{\partial}{\partial \omega_{F,t}} \left(\frac{1 - \omega_{S,t} \xi_t}{\eta \omega_{S,t} + \omega_{F,t}} \right) = \frac{\eta + \xi_t - 1}{\Lambda_t^2}. \quad (51)$$

Consequently, a larger effective FX-backing share dampens the liability elasticity if and only if

$$\xi_t < 1 - \eta. \quad (52)$$

For a fully unfunded liability shock, $\xi_t = 0$, the effective FX-backing share necessarily dampens the impact response. If $\xi_t > 1 - \eta$, a larger effective FX-backing share can increase the elasticity.

Proposition 3 separates a level effect from an elasticity effect. A larger $\mathcal{F}_{t-1}^*$ always strengthens fiscal backing and appreciates the currency, other things equal. The derivative with respect to $\omega_{F,t}$ asks a different question: how does the liability elasticity vary across regular equilibria with different backing compositions?

The share $\omega_{F,t}$ contains the equilibrium exchange rate. It is not a policy instrument. The derivative is therefore not the total effect of an operation that changes the FX position, the exchange rate, and public resources at the same time.

The sign reversal has a simple source. More FX backing raises the weight on the unit-elastic revaluation term, which tends to reduce the exchange-rate response. It also lowers the resource-backed share to which ξ_t applies. If the fiscal response is strong, the second effect can dominate. Thus FX backing improves the level of fiscal support, but it reduces the liability elasticity only when $\xi_t < 1 - \eta$. For a fully unfunded shock, $\xi_t = 0$, the elasticity necessarily falls.

Remark 5 (Generalized damping condition). *When $\chi_t \neq 0$, define $\Lambda_t^\chi \equiv \omega_{S,t}(\eta + \chi_t) + \omega_{F,t}$. Holding ξ_t and χ_t fixed across local comparisons,*

$$\frac{\partial}{\partial \omega_{F,t}} \left(\frac{1 - \omega_{S,t} \xi_t}{\Lambda_t^\chi} \right) = \frac{\eta + \chi_t + \xi_t - 1}{(\Lambda_t^\chi)^2}. \quad (53)$$

Thus, the generalized damping condition is $\xi_t < 1 - \eta - \chi_t$. Equation (52) is the exchange-rate-independent-backing benchmark requested for Proposition 3.

4.3. Private foreign assets are not public foreign assets

From eq. (17), private foreign assets affect the valuation equation only through θA_{t-1}^* .

Corollary 2 (Fiscal relevance of private foreign assets). *At a locally regular equilibrium,*

$$\frac{\partial E_t}{\partial A_{t-1}^*} = \theta \frac{\partial E_t}{\partial \mathcal{F}_{t-1}^*}. \quad (54)$$

If $\theta = 0$, private foreign assets have no direct public-backing effect. If $\theta > 0$, they strengthen public backing only through the present value of the associated tax claim.

If $A_{t-1}^* > 0$, define the critical present-value tax-claim coefficient

$$\theta_t^{\text{crit}} \equiv \frac{D_{t-1}^* - R_{t-1}^*}{A_{t-1}^*}. \quad (55)$$

When $0 < \theta_t^{\text{crit}} < 1$, the effective foreign-currency position is negative for $\theta < \theta_t^{\text{crit}}$, so a Proposition 4-type fold can arise under its other conditions; for $\theta \geq \theta_t^{\text{crit}}$, the effective position is nonnegative and that negative-position fold is absent. If $D_{t-1}^* \leq R_{t-1}^*$, the public position is nonnegative even at $\theta = 0$. If $\theta_t^{\text{crit}} > 1$, a negative effective position can remain even under the maximal unit tax claim.

For the country calculations, fix a reference exchange rate $\bar{E}$ and a common valuation basis for $\mathcal{B}$, D^* , and R^* . Define the tangency tax-claim threshold as

$$\theta_t^{\text{tan}} \equiv \frac{D_{t-1}^* - R_{t-1}^* - \frac{\eta}{1-\eta} \frac{\mathcal{B}_{t-1}}{\bar{E}}}{A_{t-1}^*}. \quad (56)$$

The tangency threshold is smaller than the sign threshold whenever $\mathcal{B}_{t-1} > 0$ and $A_{t-1}^* > 0$. If $\theta_t^{\text{tan}} \leq 0$, the reference point is already on the increasing branch when $\theta = 0$. If $0 < \theta_t^{\text{tan}} < 1$, tangency can be reached within the admissible range. If $\theta_t^{\text{tan}} > 1$, it cannot. The same endpoint logic applies to θ_t^{crit} .

Both thresholds are invariant to a consistent change in the foreign-currency unit. Equality at θ_t^{tan} places the reference point at the peak. A larger value places it on the increasing branch; a smaller value places it on the decreasing branch. These are balance-sheet thresholds, not observed tax rates or confiscation shares.

When the effective position is negative, denote the resulting effective net public foreign-currency liability by

$$\mathcal{D}_{t-1}^* \equiv -\mathcal{F}_{t-1}^* = D_{t-1}^* - R_{t-1}^* - \theta A_{t-1}^* > 0. \quad (57)$$

This object is distinct from the gross consolidated foreign-currency liability D_{t-1}^* in eq. (10). The fixed- S_t valuation bound is proportional to $(\mathcal{D}_{t-1}^*)^{-\eta/(1-\eta)}$, and hence

$$\frac{\partial \log \bar{\mathcal{B}}_t^V}{\partial \theta} = \frac{\eta}{1-\eta} \frac{A_{t-1}^*}{\mathcal{D}_{t-1}^*} > 0. \quad (58)$$

A larger credible present-value tax claim raises the valuation bound. If it is large enough, the public FX position becomes nonnegative. The private asset itself never becomes a public asset.

Remark 6 (Exchange-rate-dependent present-value tax claim). *For the fixed- S_t benchmark only, suppose $\theta = \theta(E)$, so that*

$$V(E) = S_t \Omega_t E^\eta + E [R^* - D^* + \theta(E) A^*]. \quad (59)$$

Then

$$V'(E) = \eta S_t \Omega_t E^{\eta-1} + R^* - D^* + A^* [\theta(E) + E \theta'(E)], \quad (60)$$

$$V''(E) = \eta(\eta-1) S_t \Omega_t E^{\eta-2} + A^* [2\theta'(E) + E \theta''(E)]. \quad (61)$$

Thus $\theta'(E) < 0$ lowers the marginal value of fiscal backing relative to a counterfactual that fixes the tax-claim coefficient at its current value. It does not by itself imply greater concavity, a lower bound, or a definite movement of the peak. A sufficient condition for the tax-claim term not to weaken concavity is $2\theta'(E) + E \theta''(E) \leq 0$. The main propositions hold θ fixed to preserve the closed form.

A depreciation-induced private capital gain is not a free public resource. Taxing the gain transfers wealth from private residents to the public sector. National wealth rises from depreciation only to the extent that the country as a whole has a positive net foreign-currency exposure. Appendix E separates the national resource identity from the fiscal valuation equation.

4.4. A Negative Effective Public Foreign-Currency Position and the Valuation Fold

Suppose the consolidated public sector has a negative effective foreign-currency position. Using the notation in eq. (57),

$$\mathcal{F}_{t-1}^* = -\mathcal{D}_{t-1}^*, \quad \mathcal{D}_{t-1}^* > 0. \quad (62)$$

With conditionally fixed S_t , the sticky-price valuation equation becomes

$$\mathcal{B}_{t-1} = S_t \Omega_t E_t^\eta - \mathcal{D}_{t-1}^* E_t. \quad (63)$$

Depreciation now has two opposing effects on the public balance sheet. The first term raises the CPI and lowers the real value of domestic-currency liabilities. It grows with E_t^η . The second term raises the home-currency value of the effective net foreign-currency liability. It grows one for one with E_t .

When $0 < \eta < 1$, CPI relief eventually grows more slowly than FX revaluation. The valuation schedule can therefore become hump-shaped. We call this nonlinearity a valuation fold. For the global constant-elasticity benchmark, suppose that the short-run pricing relation $P_t = \Omega_t E_t^\eta$ is extended over all $E_t > 0$.

Proposition 4 (Currency mismatch and a valuation bound). *Let $S_t > 0$, $\Omega_t > 0$, $\mathcal{D}_{t-1}^* > 0$, and $0 < \eta < 1$ be conditionally fixed. Define*

$$E_t^\dagger \equiv \left(\frac{\eta S_t \Omega_t}{\mathcal{D}_{t-1}^*} \right)^{1/(1-\eta)} \quad (64)$$

and

$$\overline{\mathcal{B}}_t^V \equiv (1-\eta) S_t \Omega_t \left(\frac{\eta S_t \Omega_t}{\mathcal{D}_{t-1}^*} \right)^{\eta/(1-\eta)}. \quad (65)$$

Then the sticky-price valuation equation has two positive exchange-rate solutions if $0 < \mathcal{B}_{t-1} < \overline{\mathcal{B}}_t^V$, one tangency solution $E_t^\dagger$ if $\mathcal{B}_{t-1} = \overline{\mathcal{B}}_t^V$, and, if $\mathcal{B}_{t-1} > \overline{\mathcal{B}}_t^V$, no positive exchange rate satisfies the maintained global constant-elasticity benchmark.

Proposition 4 gives the paper's main result. The object $\overline{\mathcal{B}}_t^V$ is the largest stock of domestic-currency liabilities that the stated balance sheet and short-run price relation can support. Liabilities below the peak have

two exchange-rate solutions. At the peak there is one. Above it there is none under the global constant-elasticity benchmark.

This valuation bound is not a long-run debt limit. Conventional debt-sustainability analysis helps determine the expected public resources summarized by S_t . The present model then asks whether those resources can support the inherited nominal liabilities at date t . If they cannot, public resources, the FX position, price setting, or the policy regime must change.

The two branches have different local slopes. On the *increasing branch*, which is the lower exchange-rate solution, $E_t < E_t^\dagger$ and

$$V_t'(E_t) = \eta S_t \Omega_t E_t^{\eta-1} - \mathcal{D}_{t-1}^* > 0. \quad (66)$$

On this branch, CPI relief rises faster at the margin than the home-currency value of the effective net FX liability. Depreciation therefore continues to reduce the valuation gap. As shown below, this is the branch connected to the no-mismatch solution.

On the *decreasing branch*, which is the higher exchange-rate solution, $E_t > E_t^\dagger$ and $V_t'(E_t) < 0$. A further depreciation now raises the home-currency value of the effective net FX liability by more than it increases CPI relief. The valuation schedule is therefore downward sloping. The model contains no adjustment law and makes no claim about the dynamic stability of either branch.

Corollary 3 (Continuation from the zero-mismatch benchmark). *Under the assumptions of Proposition 4, let $A_t \equiv S_t \Omega_t > 0$ and fix $\mathcal{B}_{t-1} > 0$. When $\mathcal{D}_{t-1}^* = 0$, the unique finite solution is*

$$E_t^0 = \left(\frac{\mathcal{B}_{t-1}}{A_t} \right)^{1/\eta}. \quad (67)$$

Whenever two positive solutions exist for $\mathcal{D}_{t-1}^ > 0$, denote them by $E_t^L(\mathcal{D}_{t-1}^*) < E_t^\dagger < E_t^U(\mathcal{D}_{t-1}^*)$. Then*

$$\lim_{\mathcal{D}_{t-1}^* \downarrow 0} E_t^L(\mathcal{D}_{t-1}^*) = E_t^0, \quad (68)$$

$$\lim_{\mathcal{D}_{t-1}^* \downarrow 0} E_t^U(\mathcal{D}_{t-1}^*) = \infty. \quad (69)$$

Thus continuity from the zero-mismatch benchmark uniquely identifies the increasing branch among finite valuation solutions.

Both branches solve the same consolidated present-value equation under the no-Ponzi restrictions. The second branch is not generated by adding a separate bubble term to the conditional valuation equation. It is a second solution to the same static present-value relation. Whether it can be supported by a dynamically consistent financial-market equilibrium lies outside the model.

Continuity provides a static refinement, not a stability criterion. With a small positive mismatch, the increasing solution remains close to the no-mismatch exchange rate, while the decreasing solution lies beyond a distant peak. As mismatch rises, the two solutions meet at tangency and disappear. Static multiplicity therefore implies neither dynamic instability nor a rational bubble.

Implications for fiscal determinacy. Proposition 4 shows that the government valuation relation and the maintained short-run pricing relation need not deliver a globally unique exchange rate when the public sector has a sufficiently negative foreign-currency position. The same inherited liabilities and fiscal backing can then be consistent with two static valuation solutions. This multiplicity does not overturn determinacy results obtained from fully specified monetary–fiscal policy regimes. Rather, it shows that the valuation relation alone is not sufficient for global uniqueness once currency mismatch makes the schedule nonmonotone.

Additional restrictions on policy behavior, expectations, and financial-market equilibrium are needed to select among the static solutions. Continuity from the zero-mismatch benchmark provides a useful static refinement, but it is not a dynamic determinacy result.

Remark 7 (Branch sensitivity near the valuation ceiling). *Holding S_t , Ω_t , and $\mathcal{D}_{t-1}^*$ fixed, implicit differentiation gives*

$$\frac{\partial E_t}{\partial \mathcal{B}_{t-1}} = \frac{1}{\eta S_t \Omega_t E_t^{\eta-1} - \mathcal{D}_{t-1}^*}. \quad (70)$$

Holding $\mathcal{B}_{t-1}$ instead fixed, public-resource news has the response

$$\frac{\partial E_t}{\partial S_t} = -\frac{\Omega_t E_t^\eta}{\eta S_t \Omega_t E_t^{\eta-1} - \mathcal{D}_{t-1}^*}. \quad (71)$$

As the tangency is approached, the common denominator tends to zero. On the increasing branch, $\partial E_t^L / \partial \mathcal{B}_{t-1} \rightarrow +\infty$ and $\partial E_t^L / \partial S_t \rightarrow -\infty$; the signs reverse on the decreasing branch. On the continuity-refined increasing branch, both a small increase in domestic-currency liabilities and a small downward revision of expected public resources can therefore generate a disproportionately large depreciation when the economy is already close to the valuation bound. This conclusion requires that the economy be on the increasing branch and that the maintained short-run pricing closure remain credible near tangency; it is not a statement that small news necessarily causes an exchange-rate crisis. The divergent comparative-static sensitivity is a generic local property of a smooth fold. The economic result specific to this paper is that public currency mismatch and incomplete CPI adjustment generate such a fold in exchange-rate valuation.

If $\mathcal{B}_{t-1} > \bar{\mathcal{B}}_t^V$, no positive exchange rate satisfies the valuation equation under the stated assumptions. At least one assumption must then change. Public resources may rise, the FX position may improve, liabilities may be restructured, price setting may change, or the policy regime may shift. The model does not choose among these adjustments.

Remark 8 (Global benchmark and local admissibility). *The constant-elasticity relation $P_t = \Omega_t E_t^\eta$ has two uses. Proposition 4 extends it over all positive exchange rates to obtain the exact global solution count. In applications, the relation is better viewed as a local approximation over an interval $E_t / \bar{E} \in [\underline{z}, \bar{z}]$. Only solutions inside that interval are then economically admissible. If no mathematical solution lies inside it, the correct statement is that no admissible solution exists under the maintained local pricing approximation; the model does not rule out solutions under a different price-setting relation outside the interval. State-dependent price resetting, margins, invoicing, and expectations can alter the schedule away from the reference point.*

4.5. Exchange-Rate-Dependent Backing

To examine whether the fold depends on conditionally fixed public resources, suppose locally and globally over the relevant range that

$$S_t(E_t) = \bar{S}_t E_t^\chi, \quad \bar{S}_t > 0. \quad (72)$$

Let

$$\gamma \equiv \eta + \chi. \quad (73)$$

Then eq. (63) generalizes to

$$\mathcal{B}_{t-1} = \bar{S}_t \Omega_t E_t^\gamma - \mathcal{D}_{t-1}^* E_t. \quad (74)$$

Proposition 5 (Robustness of the sticky-price fold). *Under eq. (72), the strict concave valuation fold survives if and only if*

$$0 < \eta + \chi < 1. \quad (75)$$

When this condition holds, the critical exchange rate and valuation bound are obtained from eqs. (64) and (65) by replacing S_t with $\bar{S}_t$ and η with $\gamma = \eta + \chi$. If $\gamma < 0$, the valuation function is strictly decreasing and convex; if $\gamma = 0$, it is linear and decreasing; if $\gamma = 1$, it is linear; and if $\gamma > 1$, it is convex. None of these cases exhibits the strict concave two-branch fold of Proposition 4.

Within $0 < \gamma < 1$, the generalized fold is nondegenerate. For $H(E, \mathcal{B}) = \bar{S}_t \Omega_t E^\gamma - \mathcal{D}_{t-1}^* E - \mathcal{B}$, the peak satisfies $H = H_E = 0$, while $H_{EE} = \bar{S}_t \Omega_t \gamma(\gamma - 1) E^{\gamma-2} \neq 0$ and $H_{\mathcal{B}} = -1 \neq 0$.

The $\gamma = 0$ case in Proposition 5 is a linear decreasing negative-position schedule. It is distinct from the loss of local valuation regularity as $\eta + \chi_t \rightarrow 0$ in the zero-position benchmark of Proposition 1.

A positive χ reduces the local impact depreciation in Proposition 1 by improving backing as the currency depreciates. At the same time, it raises the exponent γ and weakens the concavity that generates the fold. No general monotonic claim is made about how χ changes the numerical value of the valuation bound: that comparative static depends on the scale of $\bar{S}_t \Omega_t$, $\mathcal{D}_{t-1}^*$, and the normalization of E_t . Within the fold region, the exchange-rate elasticity of backing that places a given reference point at the peak is $\chi^{\text{tan}} = \delta - \eta$. Smaller values place the point on the decreasing branch; larger values place it on the increasing branch.

Remark 9 (Fold under the complete fiscal-rule specialization). *Under the complete specialization in eq. (31), direct substitution into the negative-position valuation equation gives*

$$(1 - \Phi_t) \mathcal{B}_{t-1} = \bar{S}_t \Omega_t E_t^\eta - \mathcal{D}_{t-1}^* E_t.$$

Hence, for $0 \leq \Phi_t < 1$ and $0 < \eta < 1$, the same strictly concave fold geometry survives. The fiscal reaction reduces the effective unfunded portion of inherited domestic-currency liabilities but does not eliminate the fold. This conclusion follows directly from the fiscal rule and does not require treating the local relation $\chi_t = -\eta \xi_t$ as a global constant-elasticity backing schedule.

4.6. Alternative price and financial closures

4.6.1. Sticky-price valuation closure

The sticky-price closure is eq. (23). It treats $P_{H,t}$ and Ψ_t as predetermined during the impact interval and allows E_t to provide the immediate valuation margin. Under a negative effective public foreign-currency position,

$$\mathcal{B}_{t-1} = S_t \Omega_t E_t^\eta - \mathcal{D}_{t-1}^* E_t. \quad (76)$$

The first term is proportional to E_t^η , while the effective net public foreign-currency liability is proportional to E_t . For $0 < \eta < 1$, this difference in elasticities produces the concave fold.

4.6.2. UIP–Fisher closure

Let lower-case letters denote logarithms: $e_t \equiv \log E_t$, $p_t \equiv \log P_t$, and $p_t^* \equiv \log P_t^*$. The log real exchange rate is

$$q_t \equiv e_t + p_t^* - p_t. \quad (77)$$

Log uncovered interest parity is

$$e_t = \mathbb{E}_t e_{t+1} + i_t^* - i_t + \rho_t, \quad (78)$$

where i_t and i_t^* are one-period nominal interest rates and ρ_t is the risk premium. The home and foreign Fisher relations are

$$i_t = r_t + \mathbb{E}_t \pi_{t+1}, \quad (79)$$

$$i_t^* = r_t^* + \mathbb{E}_t \pi_{t+1}^*, \quad (80)$$

where r_t and r_t^* are real interest rates and π_{t+1} and π_{t+1}^* are inflation rates. Combining eqs. (77) to (80) gives

$$q_t = \mathbb{E}_t q_{t+1} + r_t^* - r_t + \rho_t. \quad (81)$$

Combining uncovered interest parity with the home and foreign Fisher relations expresses the nominal parity condition in real terms. The UIP–Fisher closure conditions on the expected future real exchange rate, the real interest differential, and the risk premium. These objects jointly imply a conditional current real exchange rate, denoted $Q_t^{UF} \equiv \exp(q_t^{UF})$. The closure does not independently fix nominal interest rates, expected inflation rates, and real interest rates, since those variables are linked by the Fisher relations. Fiscal news may change the expected future real exchange rate, the real interest differential, or the risk premium. The present exercise conditions on the resulting current real exchange rate rather than deriving those responses. Expected inflation is not irrelevant; nominal interest and expected-inflation differentials are incorporated into the real interest differential and the expected future real exchange rate when nominal UIP is written in real terms. UIP–Fisher therefore provides a conditional real-exchange-rate closure and does not determine the nominal exchange-rate level by itself.

The consolidated valuation equation can be written

$$\mathcal{B}_{t-1} = P_t \left(S_t + \frac{Q_t}{P_t^*} \mathcal{F}_{t-1}^* \right). \quad (82)$$

If $Q_t = Q_t^{UF}$ is fixed by UIP–Fisher consistency and $S_t + (Q_t^{UF}/P_t^*) \mathcal{F}_{t-1}^* > 0$, then

$$P_t^{UF} = \frac{\mathcal{B}_{t-1}}{S_t + \frac{Q_t^{UF}}{P_t^*} \mathcal{F}_{t-1}^*}, \quad (83)$$

$$E_t^{UF} = \frac{Q_t^{UF}}{P_t^*} \frac{\mathcal{B}_{t-1}}{S_t + \frac{Q_t^{UF}}{P_t^*} \mathcal{F}_{t-1}^*}. \quad (84)$$

UIP alone does not determine the nominal exchange-rate level. UIP–Fisher consistency constrains the real exchange rate, and the fiscal valuation equation jointly determines the price level and nominal exchange-rate level.

4.6.3. PPP closure

Under a long-run PPP closure, let the real exchange rate be fixed at $\bar{Q} > 0$:

$$\frac{E_t P_t^*}{P_t} = \bar{Q}. \quad (85)$$

If $S_t + (\bar{Q}/P_t^*)\mathcal{F}_{t-1}^* > 0$, then

$$P_t^{PPP} = \frac{\mathcal{B}_{t-1}}{S_t + \frac{\bar{Q}}{P_t^*}\mathcal{F}_{t-1}^*}, \quad (86)$$

$$E_t^{PPP} = \frac{\bar{Q}}{P_t^*} \frac{\mathcal{B}_{t-1}}{S_t + \frac{\bar{Q}}{P_t^*}\mathcal{F}_{t-1}^*}. \quad (87)$$

The PPP closure is a long-run special case of the UIP–Fisher closure when the expected future real exchange rate equals its long-run value, the real interest differential vanishes, and the risk premium is zero.

4.6.4. Conditionally fixed-real-exchange-rate closures

Proposition 6 (No Two-Branch Fold under a Conditionally Fixed Real Exchange Rate and Exchange-Rate-Invariant Public Resources). *Condition on a current real exchange rate $Q_t > 0$ and conditionally fixed $S_t > 0$. If $\mathcal{F}_{t-1}^* = -\mathcal{D}_{t-1}^* < 0$, the valuation equation becomes*

$$\mathcal{B}_{t-1} = E_t \left(\frac{P_t^* S_t}{Q_t} - \mathcal{D}_{t-1}^* \right). \quad (88)$$

If $P_t^ S_t > Q_t \mathcal{D}_{t-1}^*$, there is a unique positive conditional valuation equilibrium. If $P_t^* S_t \leq Q_t \mathcal{D}_{t-1}^*$, there is no positive conditional valuation equilibrium for $\mathcal{B}_{t-1} > 0$. There are never two positive valuation branches.*

Proposition 6 does not claim that uncovered interest parity by itself eliminates valuation nonlinearities. It applies to closures under which the current real exchange rate is conditionally fixed with respect to the date- t fiscal valuation exercise. Fixing the current real exchange rate alone need not eliminate valuation nonlinearities if public resources remain exchange-rate dependent; Proposition 6 conditions on both the current real exchange rate and conditionally fixed public resources. UIP–Fisher and PPP are two examples. If fiscal news changes the expected future real exchange rate, the real interest differential, or the risk premium, the current real exchange rate can change as well; the proposition conditions on those responses. Given Q_t , $P_t = E_t P_t^*/Q_t$, so both the public-resource term and the effective net public foreign-currency liability are linear in E_t . The curvature responsible for Proposition 4 is absent. This closure dependence is economically informative: the fold is an impact phenomenon that requires domestic-price adjustment to lag behind the home-currency revaluation of the effective net public foreign-currency liability. Under the sticky-price closure, CPI-based backing moves with E_t^η while the liability is revalued one for one; under fixed Q_t and exchange-rate-invariant backing, both terms are linear in E_t .

4.7. Central-Bank Balance-Sheet Operations and Foreign-Exchange Intervention

The consolidated nominal liability is

$$\mathcal{B}_{t-1} = B_{t-1}^G - B_{t-1}^C + H_{t-1}. \quad (89)$$

Suppose first that the central bank purchases $\Delta B^C > 0$ of government bonds from the private sector and credits reserve balances by the same nominal amount, so that $\Delta H = \Delta B^C$. If no other item changes,

$$\Delta \mathcal{B} = -\Delta B^C + \Delta H = 0. \quad (90)$$

The transaction swaps one consolidated public liability for another. It can nevertheless affect S_t^C by replacing long-duration fixed-rate debt held outside the central bank with short-duration interest-bearing reserves. Domestic government-bond purchases therefore operate through liability composition and future central-bank resource income, not through the mechanical disappearance of consolidated liabilities.

Foreign-exchange intervention requires the same two-sided accounting. Let $x > 0$ be the foreign-currency amount of reserves sold during period t . Because

$$\mathcal{F}^* = R^* - D^* + \theta A^*, \quad (91)$$

the general change in fiscally usable foreign-currency backing is

$$\Delta \mathcal{F}^* = -x + \theta \Delta A^*. \quad (92)$$

The benchmark below assumes that the domestic private foreign-asset tax base is unchanged, $\Delta A^* = 0$, and therefore $\Delta \mathcal{F}^* = -x$. If domestic residents ultimately acquire the foreign currency, so that $\Delta A^* = x$, then $\Delta \mathcal{F}^* = -(1 - \theta)x$: part of the lost public reserve asset remains fiscally relevant through the present value of the tax claim on private foreign assets. More generally, the direct valuation effect depends on the intervention counterparty, the final holder of the foreign currency, and the accompanying liability and public-resource changes.

To describe an intraperiod intervention without rewriting inherited opening positions, let $(\mathcal{B}_t^-, \mathcal{F}_t^{*, -}, E_t^-)$ denote the valuation immediately before the transaction and $(\mathcal{B}_t^+, \mathcal{F}_t^{*, +}, E_t^+)$ the valuation immediately after it. In the benchmark case $\Delta A^* = 0$, if the home-currency proceeds received by the central bank are retired, then

$$\mathcal{B}_t^+ = \mathcal{B}_t^- - E_t^- x, \quad (93)$$

$$\mathcal{F}_t^{*, +} = \mathcal{F}_t^{*, -} - x. \quad (94)$$

Suppose the pre-intervention valuation satisfies

$$\mathcal{B}_t^- = S_t \Omega_t (E_t^-)^\eta + E_t^- \mathcal{F}_t^{*, -}. \quad (95)$$

Using eqs. (93) and (94),

$$\mathcal{B}_t^+ = \mathcal{B}_t^- - E_t^- x \quad (96)$$

$$= S_t \Omega_t (E_t^-)^\eta + E_t^- (\mathcal{F}_t^{*, -} - x) \quad (97)$$

$$= S_t \Omega_t (E_t^-)^\eta + E_t^- \mathcal{F}_t^{*, +}. \quad (98)$$

Thus E_t^- remains a solution of the post-intervention valuation equation. In the local policy exercise conducted on the same regular branch, $E_t^+ = E_t^-$. The locally upward-sloping branch used here is also the branch continuously connected to the unique zero-mismatch valuation solution. This continuity property provides a natural refinement for the local policy exercise, although it does not establish a dynamic selection result.

Remark 10 (Current-market-value neutrality of a matched reserve sale). *A reserve sale matched by an equal current-market-value reduction in consolidated home-currency liabilities is directly neutral in the date- t valuation equation, holding expected public resources and the price closure fixed. The public sector gives up a foreign-currency asset and retires a home-currency liability of the same transaction value. The result establishes neutrality of the contemporaneous balance-sheet revaluation at transaction values. It does not establish neutrality of the full fiscal valuation once the operation changes future foreign-asset income, reserve remuneration, seigniorage, remittances, or other components of expected public resources. Nor does it imply that observed intervention is ineffective: order flow, portfolio balance, liquidity, policy signaling, expected*

future exchange rates, and risk premia remain outside the direct valuation channel isolated here.

For completeness, the local differential version of the result follows from

$$d\mathcal{B}_t = \left(\eta S_t \Omega_t E_t^{\eta-1} + \mathcal{F}_t^* \right) dE_t + E_t d\mathcal{F}_t^*. \quad (99)$$

The matched transaction satisfies $d\mathcal{B}_t = E_t d\mathcal{F}_t^*$, and hence $dE_t = 0$ at a locally regular point.

The term “sterilized intervention” requires care in consolidated accounting. Suppose that, after selling reserves and reducing H_t by $E_t^- x$, the central bank purchases $E_t^- x$ of government bonds from the private sector to restore the monetary base. The sterilizing purchase itself satisfies $\Delta B_t^C = \Delta H_t = E_t^- x$ and therefore contributes

$$-\Delta B_t^C + \Delta H_t = 0 \quad (100)$$

to consolidated nominal liabilities. The combined reserve sale and government-bond purchase still reduces $\mathcal{B}_t$ by $E_t^- x$ at transaction values. The domestic purchase changes the composition of public liabilities rather than mechanically reversing the consolidated liability retirement. Its fiscal consequences operate through maturity, reserve remuneration, asset income, remittances, and future S_t^C . If sterilization instead involves private domestic assets, those assets and their future returns must be included explicitly before the consolidated valuation effect can be assessed.

For comparison, retain the benchmark $dA^* = 0$ but hold consolidated domestic-currency liabilities fixed locally:

$$d\mathcal{F}_t^* = -dx, \quad d\mathcal{B}_t = 0. \quad (101)$$

This is not the matched reserve-sale transaction above. It requires new net public liability issuance or another operation that offsets the liability retirement. From eq. (99),

$$dE_t = \frac{E_t dx}{\eta S_t \Omega_t E_t^{\eta-1} + \mathcal{F}_t^*}. \quad (102)$$

On the continuity-refined increasing branch, the denominator is positive, so unmatched reserve depletion raises fiscal valuation pressure toward depreciation. The expression is not used to infer policy effects on the decreasing branch or beyond the valuation bound.

The same distinction can be expressed in terms of an exchange-rate objective. The relevant fiscal-policy question is not whether a reserve sale is neutral at transaction values, but whether the operation changes the expected public resources required to support the exchange-rate objective. Under the sticky-price closure, public resources required to support a target $\bar{E} > 0$ are

$$S_t^{\text{req}}(\bar{E}) = \frac{\mathcal{B}_t - \bar{E} \mathcal{F}_t^*}{\Omega_t \bar{E}^\eta}. \quad (103)$$

For a negative effective position, $\mathcal{F}_t^* = -\mathcal{D}_t^*$, the numerator is equivalently $\mathcal{B}_t + \bar{E} \mathcal{D}_t^*$. For a matched benchmark reserve sale at $\bar{E}$, $\Delta \mathcal{B}_t = \bar{E} \Delta \mathcal{F}_t^*$, so $\mathcal{B}_t - \bar{E} \mathcal{F}_t^*$ and therefore $S_t^{\text{req}}(\bar{E})$ are unchanged. The operation does not by itself imply a need for future tax increases. If instead $\Delta \mathcal{F}_t^* = -x$ and $\Delta \mathcal{B}_t = 0$, required resources rise by $\bar{E} x / (\Omega_t \bar{E}^\eta) > 0$. Maintaining the same exchange rate then requires higher expected taxes, lower spending, improved central-bank earnings, or another increase in public resources.

Reserve sales also need not move the economy mechanically closer to the valuation bound. While $\mathcal{F}_t^* > 0$, the fold in Proposition 4 does not exist. If the position is negative, $\mathcal{F}_t^* = -\mathcal{D}_t^*$, holding current

domestic-currency liabilities fixed gives

$$\frac{d \log \bar{\mathcal{B}}_t^V}{d \log \mathcal{D}_t^*} = -\frac{\eta}{1 - \eta} < 0. \quad (104)$$

A matched reserve sale, however, also retires current liabilities. Whether the liability position moves closer to or farther from the revised ceiling depends on the relative sizes of the ceiling change and liability retirement and is not signed in general.

Table 1 summarizes the direct consolidated-valuation effects. The entries exclude intervention channels operating through market demand, liquidity, signaling, expected future exchange rates, and risk premia.

Table 1: Foreign-exchange intervention and consolidated valuation

Operation	Change in $\mathcal{F}^*$	Change in $\mathcal{B}$	Direct fiscal valuation implication
Reserve sale with liability retirement; private foreign-asset base unchanged	$-x$	$-E_t^- x$	Directly neutral at current transaction values, conditional on fixed S_t and price closure.
Reserve sale to domestic residents, $\Delta A^* = x$	$-(1 - \theta)x$	Transaction-dependent	Part of the reserve loss remains as a tax claim; no unconditional sign without the complete liability transaction.
Reserve sale followed by a government-bond purchase restoring the monetary base	As determined by final foreign holder	$-E_t^- x$ in the benchmark	The bond purchase changes liability composition; future S_t^C may change.
Reserve depletion with consolidated liabilities held fixed	$-x$ in the benchmark	0	Depreciation pressure on the continuity-refined increasing branch.
Exchange-rate target accompanied by an increase in public resources	Transaction-dependent	Transaction-dependent	Feasibility is governed by $S_t^{\text{req}}(\bar{E})$, not by reserve loss alone.

Note: $x > 0$ is measured in foreign-currency units. The table reports only the direct consolidated fiscal valuation channel. Actual intervention can also operate through order flow, portfolio composition, liquidity, signaling, expected future exchange rates, and risk premia.

The policy implication is conditional. When fiscal under-backing is an important source of depreciation, intervention alone does not eliminate the underlying valuation gap. A reserve sale matched by an equal current-market-value liability retirement is directly valuation-neutral in the consolidated equation. Unmatched reserve depletion weakens backing on the increasing branch. Sustained exchange-rate support requires consistency among public foreign assets, consolidated nominal liabilities, expected public resources, and market expectations.

4.8. Distance to the Valuation Fold

The distance calculation extends naturally to exchange-rate-dependent fiscal backing. Suppress date subscripts and write the negative-position schedule as

$$V(E) = \bar{S}\Omega E^\gamma - \mathcal{D}^* E, \quad \gamma = \eta + \chi.$$

A fold can arise only when $0 < \gamma < 1$. At a reference exchange rate $\bar{E} > 0$, define

$$\delta \equiv \frac{\mathcal{D}^* \bar{E}}{\bar{S} \Omega \bar{E}^\gamma} = \frac{\mathcal{D}^* \bar{E}}{\mathcal{B} + \mathcal{D}^* \bar{E}}. \quad (105)$$

The second equality uses the valuation equation at the reference point. The ratio is dimensionless. It measures the share of gross backing at the reference point absorbed by the effective net public foreign-currency liability.

The local slope is

$$V'(\bar{E}) = \bar{S} \Omega \bar{E}^{\gamma-1} (\gamma - \delta). \quad (106)$$

For $0 < \gamma < 1$, define the generalized proximity ratio

$$r_\chi \equiv \frac{\delta}{\gamma} > 0. \quad (107)$$

The reference point is on the increasing branch when $0 < r_\chi < 1$, at the peak when $r_\chi = 1$, and on the decreasing branch when $r_\chi > 1$. The corresponding backing-elasticity threshold is

$$\chi^{\text{tan}} = \delta - \eta,$$

interpreted only within $0 < \eta + \chi < 1$.

The peak satisfies

$$\frac{E^\dagger}{\bar{E}} = \left(\frac{\gamma}{\delta} \right)^{1/(1-\gamma)} = r_\chi^{-1/(1-\gamma)}. \quad (108)$$

A signed logarithmic distance is

$$d_{\text{fold}} \equiv \log \left(\frac{E^\dagger}{\bar{E}} \right) = -\frac{\log r_\chi}{1-\gamma}.$$

A positive value places the peak at a higher exchange rate than the reference point. Zero means the reference point is at the peak. A negative value means that the peak lies below the reference exchange rate and the reference point is already on the decreasing branch. The phrase “remaining depreciation distance” is used only when $d_{\text{fold}} > 0$.

Under fixed public resources, $\chi = 0$, $\gamma = \eta$, and the benchmark ratio becomes

$$r \equiv \frac{\delta}{\eta} > 0, \quad d_{\text{fold}} = -\frac{\log r}{1-\eta}.$$

Thus $0 < r < 1$ identifies the increasing branch, $r = 1$ the peak, and $r > 1$ the decreasing branch. Conditional on r ,

$$\frac{\partial d_{\text{fold}}}{\partial \eta} = \frac{-\log r}{(1-\eta)^2}. \quad (109)$$

The derivative is positive on the increasing branch and negative on the decreasing branch. In either case, a lower η moves the signed distance toward zero conditional on r ; it does not rank countries because δ also varies with their balance sheets.

Let the local pass-through approximation be maintained only for $E/\bar{E} \in [\underline{z}, \bar{z}]$. A mathematical solution or peak is economically admissible only if its exchange-rate ratio lies in that interval. For a peak to the right of the reference point, $E^\dagger/\bar{E} \leq \bar{z}$ is equivalent to

$$\frac{\delta}{\eta} \geq \bar{z}^{-(1-\eta)} \quad (110)$$

under fixed backing. An analogous lower-bound condition applies when the peak lies to the left. The values $\bar{z} = 1.10$ and $\bar{z} = 1.20$ are transparent sensitivity markers rather than estimated structural validity bounds.

Table 2: Generalized normalized distance to the valuation peak

<i>Panel A. Peak exchange rate relative to the reference rate under fixed backing</i>				
$r = \delta/\eta$	$\eta = 0.08$	$\eta = 0.15$	$\eta = 0.25$	Local classification
0.50	2.12	2.26	2.52	Increasing branch
0.75	1.37	1.40	1.47	Increasing branch
0.90	1.12	1.13	1.15	Increasing branch
1.00	1.00	1.00	1.00	Peak
1.10	0.90	0.89	0.88	Decreasing branch
1.80	0.53	0.50	0.46	Decreasing branch
<i>Panel B. Signed log distance at $\eta = 0.25$</i>				
r	$d_{\text{fold}} = -\log r/(1-\eta)$			Interpretation
0.90	0.140			Peak above reference rate
1.00	0.000			Reference point at peak
1.10	−0.127			Peak below reference rate
1.80	−0.784			Reference point beyond peak
<i>Panel C. Minimum proximity for a right-side peak inside an assumed pricing range</i>				
η	$\bar{z} = 1.10$	$\bar{z} = 1.20$		
0.08	0.916	0.846		
0.15	0.922	0.856		
0.25	0.931	0.872		

Note: Panels A and B use the fixed-backing case of eq. (108). Panel C uses eq. (110). Ratios below one place the peak above the reference exchange rate; ratios above one place put the peak below it. Only the former are future depreciation distances. The pricing ranges are scenario markers, not estimated breakpoints.

The table imposes a useful discipline. A fold that becomes relevant inside a narrow pricing interval requires the reference point to be close to the peak in normalized terms. Low effective pass-through alone does not imply proximity.

4.9. Illustrative Numerical Examples

The numerical examples display mechanisms rather than estimate a country. Table 3 reports selected benchmarks; Figures 1 and 4 vary parameters as stated in their notes.

Table 3: Selected illustrative benchmark values

Parameter	Interpretation	Illustrative values
(α, λ, η)	Import share, pass-through, and $\eta = \alpha\lambda$	(0.20, 0.40, 0.08); (0.25, 0.60, 0.15); (0.25, 1.00, 0.25)
ξ	Liability elasticity of public resources	0, 0.25, 0.50
χ	Exchange-rate elasticity of public resources	-0.05, 0.20, 0.55, 0.80
$\mathcal{F}^*$	Public foreign-currency fiscal position	negative, zero, positive

Note: Each displayed triple satisfies $\eta = \alpha\lambda$. The values are stylized and are not empirical moments or country-specific structural estimates.

Figure 1 reports the local liability elasticity $(1 - \xi)/(\eta + \chi)$ for $\chi = 0$. It avoids interpreting large finite depreciations through a closure intended for a local short-run pricing interval. Lower effective pass-through produces a larger local exchange-rate response, while a stronger fiscal response reduces it. Large local elasticities should not be mechanically extrapolated to large finite liability shocks outside the maintained pricing interval.

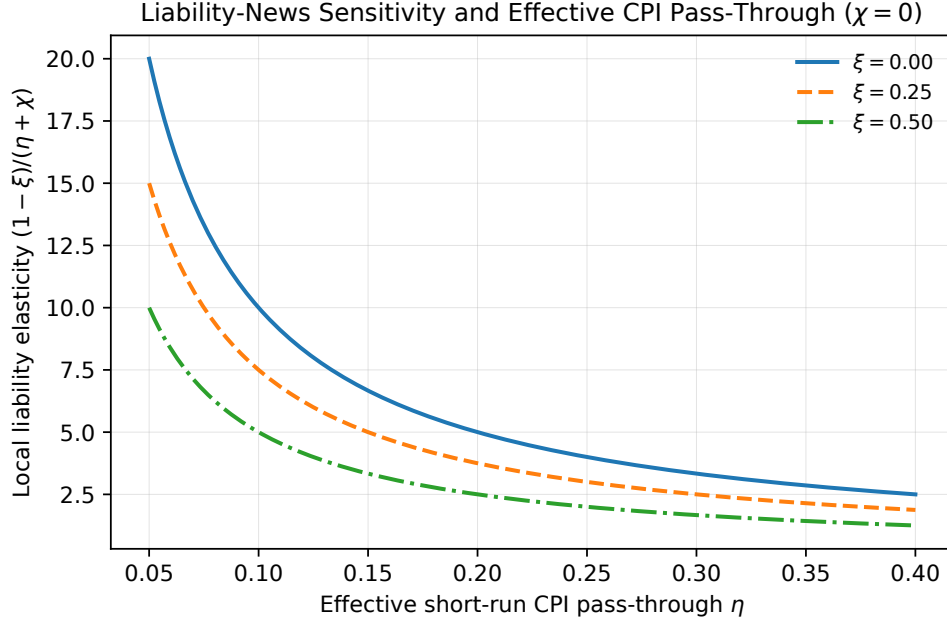


Figure 1: Liability-News Sensitivity and Effective CPI Pass-Through. *Notes:* The vertical axis is $(1 - \xi)/(\eta + \chi)$, with $\chi = 0$, $\mathcal{F}^* = 0$, $\eta \in [0.05, 0.40]$, and $\xi \in \{0, 0.25, 0.50\}$. The figure displays a local elasticity, not a finite-shock forecast. Large elasticities should not be extrapolated mechanically to large liability shocks outside the local pricing interval.

Figure 2 compares valuation-capacity functions for positive, zero, and negative effective public foreign-currency positions. A positive position steepens the schedule at high exchange rates. A negative position eventually turns the schedule downward.

Figure 3 displays the valuation fold under a tangency normalization. A liability level below the maximum intersects the schedule twice, the maximum is a tangency, and a higher liability cannot be supported by any positive exchange rate under the maintained date- t closure. The figure has economic content only to the extent that the critical exchange rate and displayed branches remain within the range in which the short-run pass-through approximation is plausible.

Figure 4 varies χ while holding the direct CPI elasticity fixed. A higher χ increases the combined exponent $\eta + \chi$, reduces the local impact response, and progressively weakens the concavity of the valuation schedule. At a unit combined exponent, the schedule becomes linear.

Figure 5 compares the three closures. The sticky-price schedule is nonlinear under a negative position, while the UIP-Fisher and PPP schedules are linear because the real exchange rate is conditionally fixed.

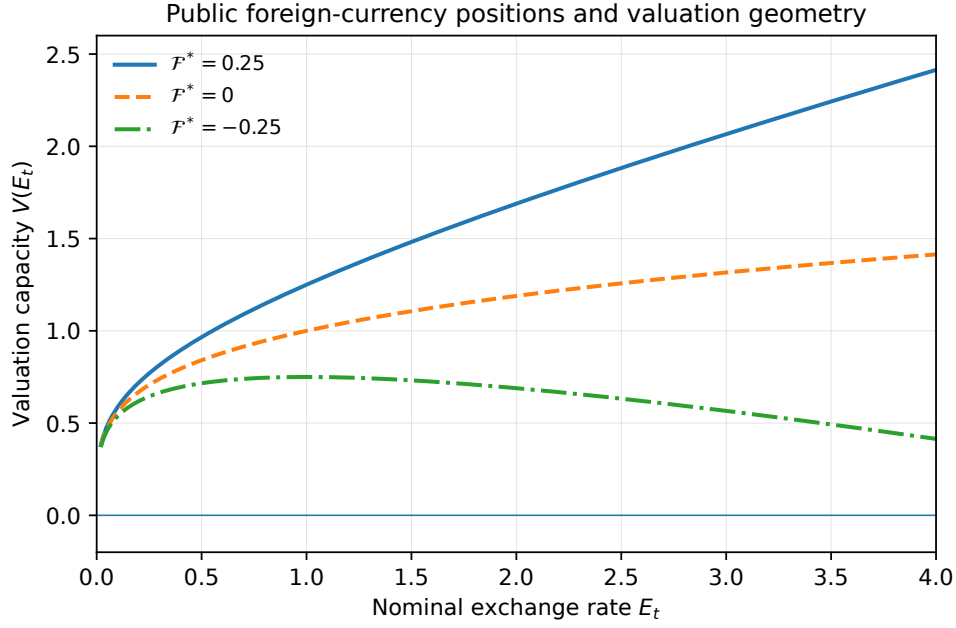


Figure 2: Public FX Positions and the Valuation Schedule. *Notes:* $S_t = 1$, $\Omega_t = 1$, $\eta = 0.25$, $\chi = \xi = 0$, $\mathcal{F}^* \in \{0.25, 0, -0.25\}$ in foreign-currency units, and E_t is normalized to one at the reference point.

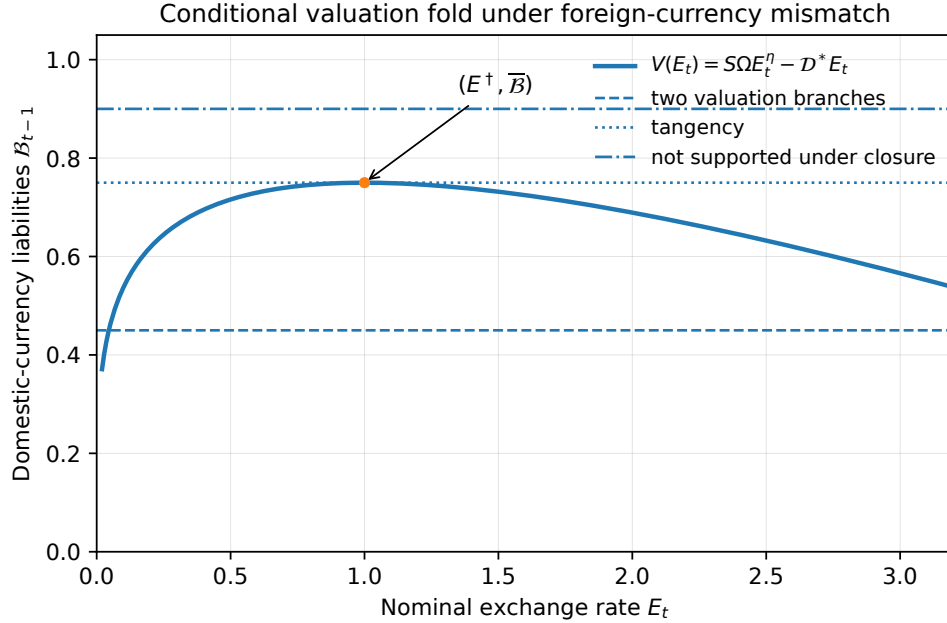


Figure 3: The Valuation Fold. *Notes:* $S_t = 1$, $\Omega_t = 1$, $\eta = 0.25$, $\chi = \xi = 0$, $\mathcal{D}^* = 0.25$ in foreign-currency units, and exchange-rate units are normalized so that the tangency occurs at $E_t^\dagger = 1$. Horizontal liability lines are 0.45, the model-implied valuation bound, and 0.90. The fold is relevant only within the range in which the short-run pass-through closure is plausible.

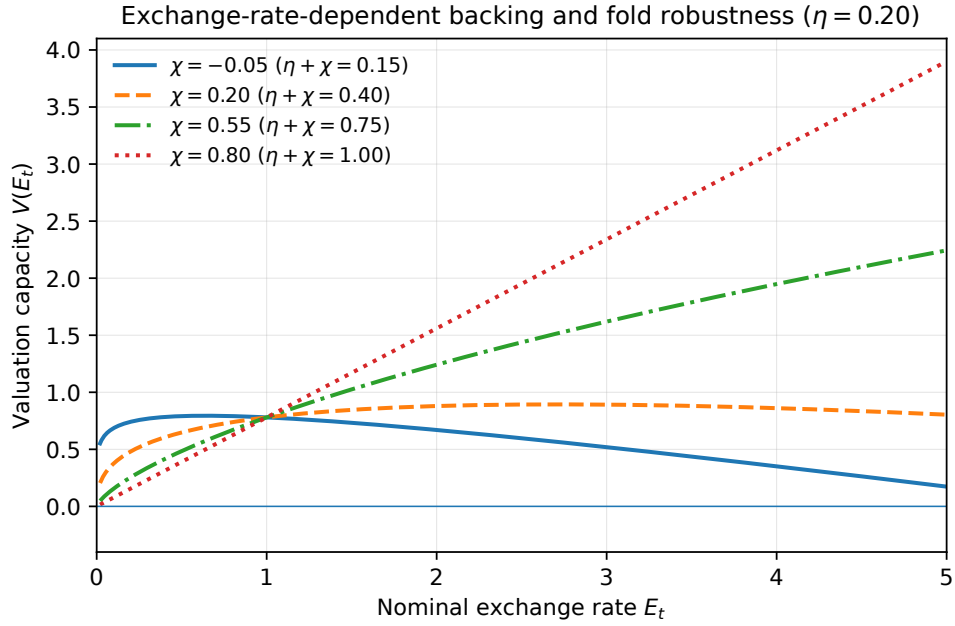


Figure 4: Exchange-Rate-Dependent Backing. *Notes:* $\bar{S}_t = 1$, $\Omega_t = 1$, $\eta = 0.20$, $\mathcal{D}^* = 0.22$ in foreign-currency units, $\xi = 0$, $\chi \in \{-0.05, 0.20, 0.55, 0.80\}$, and E_t is normalized to one at the reference point.

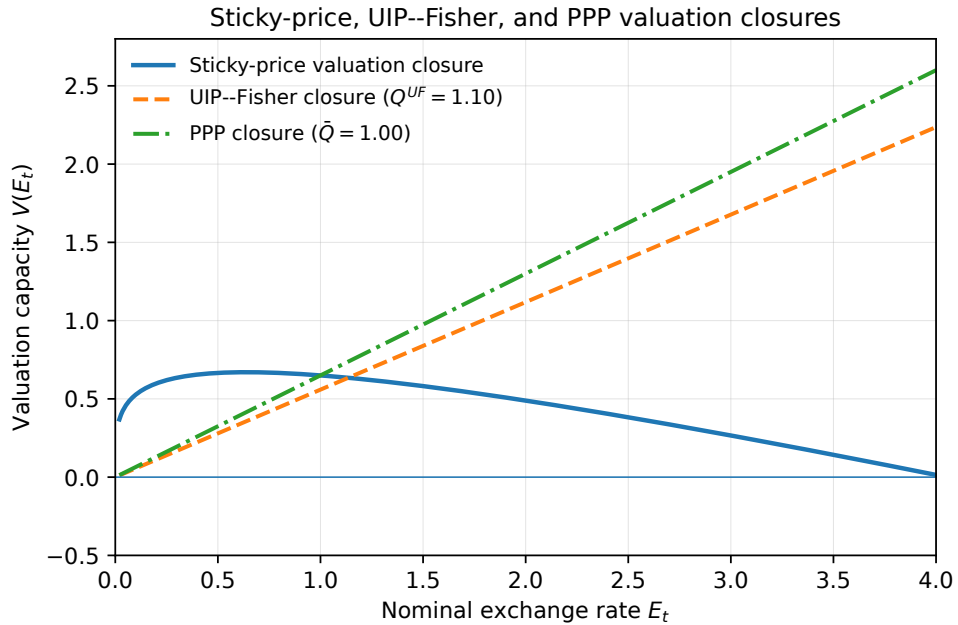


Figure 5: Valuation under Alternative Price and Financial Assumptions. *Notes:* $S_t = 1$, $\Omega_t = 1$, $\eta = 0.25$, $\chi = \xi = 0$, $\mathcal{D}^* = 0.35$ in foreign-currency units, $P_t^* = 1$, $Q_t^{UF} = 1.10$, $\bar{Q} = 1$, and E_t is normalized to one at the reference point.

Table 4: Public FX Positions and Benchmark Valuation Geometry

Effective FX position	Valuation geometry	Fiscal effect of depreciation	Main implication
$\mathcal{F}^* > 0$	Increasing; unique when $\eta + \chi > 0$	Revaluation of the positive effective public FX position raises backing	Level backing improves; elasticity damping requires a condition
$\mathcal{F}^* = 0$	Monotone; unique	CPI adjustment reduces the real burden of domestic-currency liabilities	Pass-through and fiscal backing determine impact elasticity
$\mathcal{F}^* < 0$	Single-peaked when $0 < \eta + \chi < 1$	Depreciation initially helps, but liability revaluation eventually dominates	Ceiling, two branches, and distance-to-fold arise under the stated conditions

4.10. Interpretation and empirical implications

The date- t results imply a disciplined empirical classification. Fiscal news should be separated into changes in inherited nominal liabilities and revisions to expected public resources; gross issuance alone is not an unfunded shock. Effective CPI pass-through must be measured separately from the import share, and fiscally usable foreign-currency backing must be constructed from consolidated public positions and defensible tax claims on private foreign assets.

Three distinct implications should not be conflated. First, in the zero-position benchmark, a lower effective CPI pass-through raises the local exchange-rate sensitivity to fiscal news. Second, with a negative effective position and fixed $r = \delta/\eta > 0$, a lower η reduces the absolute signed distance to the peak. Third, the curvature of the valuation schedule is not generally monotone in effective CPI pass-through: under the constant-elasticity schedule, its magnitude depends on $\gamma(1 - \gamma)$ as well as scale and the exchange-rate point. Empirical work should therefore distinguish larger local sensitivity, smaller absolute signed distance to the peak, and the existence of a single-peaked schedule rather than referring generically to “stronger nonlinearity.”

A short-run fold is most plausible when the effective public FX position is negative, normalized proximity is high, price resetting is limited, and depreciation does not raise public resources enough to offset effective-net-liability revaluation. It weakens or disappears when pass-through rises materially with depreciation, prices reset rapidly, public resources improve strongly, or a fixed-real-exchange-rate and fixed-backing closure is appropriate. The 10 and 20 percent episode thresholds used in the pass-through literature are empirically informed scenario markers, not estimates of the closure-validity radius $\bar{z}$ and not country crisis probabilities.

5. Balance-Sheet Illustrations: Argentina and Japan

5.1. Measurement

We use the country data to describe public balance sheets, not to estimate the causes of a crisis. For each country, we report the reference date, the public entities covered, the holder or residency basis, the currency denomination, and the valuation basis. We consolidate the fiscal authority and the central bank when the data allow it, and we do not count central-bank holdings of government debt twice. We report missing currency or market-value information as uncertainty rather than filling the gap with a point estimate.

The baseline country calculations hold public resources fixed, so $\chi = 0$. Proposition 5 shows how exchange-rate-dependent backing changes the relevant exponent. For Argentina, we also vary χ directly to identify the backing elasticity required to overturn the fixed-backing classification.

For the applications, let D_{gross}^* denote reported gross public FX liabilities, R_{reserve}^* official reserves, $R_{\text{other external}}^*$ other public external assets, and A_{external} a private external-asset proxy. The measured external

assets need not be wholly denominated in foreign currency. We therefore use

$$R_{\text{FX}}^* = R_{\text{reserve}}^* + \kappa_R R_{\text{other external}}^*, \quad 0 \leq \kappa_R \leq 1,$$

$$A_{\text{FX}}^* = \kappa_A A_{\text{external}}, \quad 0 \leq \kappa_A \leq 1.$$

The aggregate private external-asset measure separately measures neither θ nor κ_A . Only their product $\tilde{\theta} = \theta \kappa_A$ enters the calculation. The effective net public FX liability is

$$N_{\text{FX}}^*(\theta, \kappa_A, \kappa_R) = D_{\text{gross}}^* - R_{\text{FX}}^* - \theta \kappa_A A_{\text{external}}. \quad (111)$$

When this amount is positive, define

$$\delta(\theta, \kappa_A, \kappa_R) = \frac{\bar{E} N_{\text{FX}}^*(\theta, \kappa_A, \kappa_R)}{\mathcal{B} + \bar{E} N_{\text{FX}}^*(\theta, \kappa_A, \kappa_R)}. \quad (112)$$

The ratio δ is the share of gross backing generated through CPI adjustment that is absorbed by the effective net FX liability at the reference exchange rate. It is not a debt ratio or a crisis probability. We use $r = \delta/\eta$ to read the local slope. The schedule is increasing when $r < 1$, at its peak when $r = 1$, and decreasing when $r > 1$. If $r > 1$, the value $E^\dagger/\bar{E} < 1$ locates the model's peak under a global constant-elasticity extrapolation; it is not a future depreciation distance.

The tax-claim thresholds hold the inherited balance sheet and reference exchange rate fixed. The coefficient θ^{tan} places the reference point at the peak. A strict increase above it moves the point to the increasing branch. The larger coefficient θ^{crit} makes the public FX position nonnegative. Lower values of κ_A or κ_R raise the tax claim required to change the geometry.

The theory calls for inherited market values. The Argentina calculations instead use an illustrative exercise based on contractual exposures because the available data cannot be aligned by security, currency, issuer, holder, and market value. The Japan calculation is an accounting benchmark rather than a duration-adjusted market-value balance sheet. Table A8 documents the data mapping.

The second expression for $\delta = \bar{E} \mathcal{D}^*/(\mathcal{B} + \bar{E} \mathcal{D}^*)$ uses the valuation equation to eliminate unobserved fiscal backing. The resulting statistic is a model-implied balance-sheet diagnostic, not an independent empirical estimate of expected public resources. The calculation asks what the reported stocks imply if the reference point satisfies the model's valuation equation.

5.2. Argentina: balance-sheet stress and tax-claim thresholds

Argentina provides the negative-mismatch case. Table 5 summarizes the exchange-rate regimes and the price-setting environment. It does not claim that a fold existed in every period. High and hyperinflation are poor settings for the local price relation because firms reset prices rapidly and indexation is widespread.

Convertibility fixed the nominal exchange rate, not the real exchange rate. The exchange rate was therefore unavailable as a valuation margin before exit. We hold the inherited pre-exit balance sheet fixed and ask how the valuation schedule looks once the nominal exchange rate is allowed to move.

Official data report public external debt, reserves, other public external assets, and private nonfinancial external assets at end-2000 and end-2001 (Instituto Nacional de Estadística y Censos, 2001, 2002). The IMF reports total public debt of about US\$144.5 billion and US\$167.8 billion (International Monetary Fund, 2004).

These data do not give a security-level currency decomposition. External debt is classified mainly by creditor residence, while the model classifies liabilities by currency. The measure may omit FX debt held by residents and may include peso debt held by nonresidents. The direction of the error is therefore unknown.

Table 5: Historical regime chronology for Argentina

Period	Exchange-rate regime	Price adjustment	Public FX exposure	Model relevance
1900–1929	Gold-standard and convertibility episodes	Historical evidence only	Data incomplete	Institutional background
1930s–1945	Departure from gold and exchange controls	Regime transition	Data incomplete	Qualitative background
1946–1975	Fiscal and monetary instability	Public resources highly endogenous	Uncertain	Benchmark assumptions weak
1975–1990	High and hyperinflation	Rapid resetting, indexation, and pricing dollarization	High uncertainty	Stable short-run closure unreliable
1991–2001	Nominal convertibility peg	Nominal exchange rate unavailable as valuation margin	FX exposure documented	Conditional pre-exit stress
2001–02	Peg exit, default, and pesification	Regime and contractual adjustment	High mismatch	Realized adjustment outside model
2003–2015	Managed rates and controls	Mixed closure	Varying	Secondary background
2018–19	Flexible depreciation and IMF program	State-dependent pass-through relevant	Official FX lending important	Supplementary measurement case
2020 onward	Multiple rates and policy transitions	Closure unstable	Measurement difficult	Brief epilogue

Note: The chronology organizes regimes and measurement conditions; it does not infer a fold or the sign of the effective public FX position for every period. Sources include Daseking et al. (2005) and Independent Evaluation Office of the International Monetary Fund (2004) and official Argentine statistical publications.

Table 6 reports an illustrative exercise based on contractual exposures. It is not a market-value estimate. The residual debt measure is also not a direct observation of consolidated domestic-currency liabilities. It can contain currency, holder, issuer, and consolidation errors. The calculation shows what the available stocks imply under stated assumptions; it does not identify Argentina’s historical equilibrium branch.

The benchmark result is immediate. The value of r is about 1.8 in both years. The valuation schedule is therefore downward sloping at the reference exchange rate: FX revaluation dominates CPI relief at the margin.

The tax-claim thresholds show how much future public revenue from private foreign assets would be required to change this local geometry. Equality at θ^{tan} places the reference point at the peak. Only a larger value moves it to the increasing branch. Figure 6 reports these scenarios for $\kappa_A = 1$. Smaller foreign-currency shares of private assets raise both thresholds. The Appendix reports the full sensitivity grids.

Pass-through determines the local slope. With the balance sheet and tax claim fixed, the peak occurs when $\eta = \delta(\theta)$. At zero tax claims, the reported end-year data imply δ of about 0.45. Smaller CPI elasticities place the reference point on the decreasing part of the schedule; larger elasticities place it on the increasing part.

Exchange-rate-dependent fiscal backing changes this classification through $\gamma = \eta + \chi$. At $\eta = 0.25$, the implied peak thresholds are $\chi^{\text{tan}} = 0.205$ in 2000 and 0.201 in 2001. The fixed-backing benchmark therefore remains on the decreasing branch for moderately negative or small positive values of χ , but the classification reverses once the backing elasticity rises above about 0.20. Table A2 reports the full sensitivity grid. The calculation identifies the value of χ required to overturn the benchmark; it does not estimate χ from the data.

The calculation applies alternative local CPI elasticities to the inherited pre-exit balance sheet. It does not assume that price setting remained unchanged after exit. Pass-through, indexation, invoicing, expectations, and administered prices can all change during a regime break. The calculation therefore describes the valuation schedule under stated price assumptions. It does not estimate the realized exchange-rate or inflation path.

Official reserves fell sharply during 2001, but the normalized measure remained close to 1.8 at both year-end dates. A consistent mid-year debt measure is unavailable. Figure 7 therefore shows two end-year observations and a mid-year interval, with no line connecting them.

Table 6: Argentina: benchmark balance-sheet exercise

Panel A. Reported inputs (US\$ billions)			
Item	End-2000	End-2001	Measurement note
Total public debt	144.500	167.800	IMF public-debt table; contractual/face basis
Public external-debt proxy	84.943	88.250	Residency-based exposure proxy, not currency decomposition
Official reserves	26.917	14.913	Official reserve stock
Other public external assets	8.219	7.907	Currency share mapped with κ_R
Private external-asset proxy	98.870	101.874	Not fully identified by currency; mapped with κ_A
Reference exchange rate	1.000	1.000	Convertibility normalization
Institutional/valuation basis	Reported public-sector/external perimeters		Contractual-exposure stress; not market value
Panel B. Zero-tax-claim stress outputs under fixed public resources ($\eta = 0.25$, $\chi = 0$, $\kappa_A = 1$, $\kappa_R = 1$)			
Output	End-2000	End-2001	Interpretation
Residual nonexternal-debt proxy	59.557	79.550	Not direct consolidated $\mathcal{B}$ measurement
$N_{\text{FX}}^*(0)$	49.807	65.430	Negative-position proxy
$\delta(0)$	0.455	0.451	Share of gross CPI-based backing
$r(0) = \delta(0)/\eta$	1.822	1.805	Downward-sloping schedule at the reference point
$E^\dagger/\bar{E}$	0.449	0.455	Global closure extrapolation, not realized path
θ^{tan}	0.303	0.382	Tangency threshold; strict exceedance moves to increasing branch
θ^{crit}	0.504	0.642	Eliminates negative effective position
Panel C. Sensitivity design			
Dimension	Scenario grid		Purpose
Effective CPI pass-through	0.10, 0.15, 0.20, 0.25, 0.30, 0.40, 0.50		Local slope and $\eta^{\text{tan}} = \delta$
Present-value tax claim	0, 0.05, 0.10, 0.25, 0.50		Changes in the effective net FX liability
Private-asset FX share	$\kappa_A = 0.25, 0.50, 0.75, 1.00$		Maps private external assets into foreign-currency assets
Public-asset FX share	$\kappa_R = 0, 0.50, 1.00$		Maps other public external assets into foreign-currency assets
Currency reclassification	−20, −10, 0, 10, 20 percent of proxy		Two-sided residency-to-currency mapping uncertainty

Note: The baseline uses fixed public resources, $\chi = 0$, $\kappa_A = 1$, and $\kappa_R = 1$. The reported balance-sheet components imply $r(0) > 1$ in both end-year observations under fixed backing. The valuation schedule is locally decreasing. This is not a near-peak estimate or evidence that Argentina historically occupied a model-generated decreasing branch.

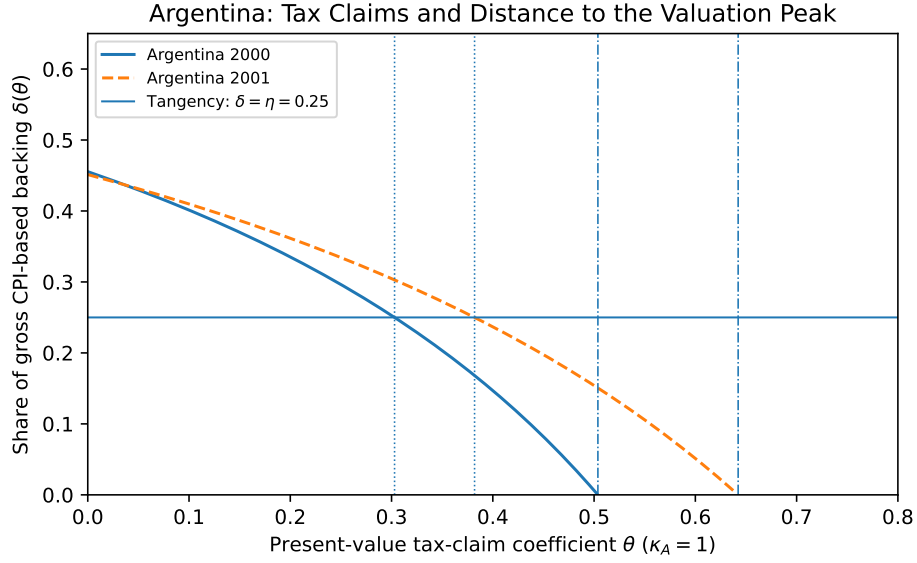


Figure 6: Argentina: Tax Claims and Distance to the Valuation Peak. The curves use the contractual-exposure stress screening in Table 6 and set $\chi = 0$, $\kappa_A = 1$, and $\kappa_R = 1$. Vertical dotted lines indicate the tax-claim coefficients that place the reference point at the peak; strict exceedance moves the reference point to the increasing branch. Dash-dotted lines indicate the sign thresholds. The figure reports scenarios, not an estimate of the observed tax claim.

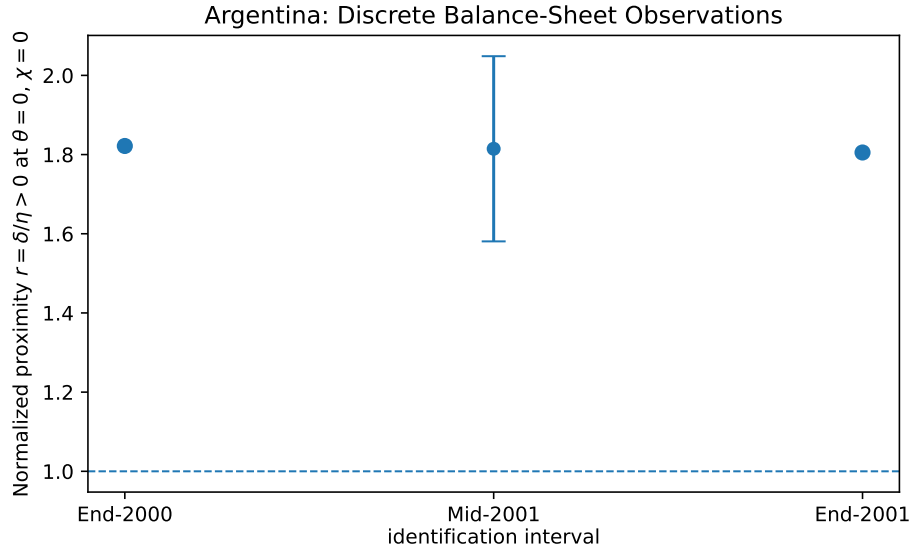


Figure 7: Argentina: Discrete Balance-Sheet Observations under $\chi = 0$, $\kappa_A = 1$, and $\kappa_R = 1$. The middle observation is an interval that combines official mid-2001 reserves with end-year debt bounds. It is not an observed mid-year balance-sheet point or a continuous time series. The figure does not establish monotonic movement toward tangency.

5.3. Argentina in 2018–19

The 2018–19 episode illustrates a different measurement problem. IMF lending can raise expected public resources, but it also increases gross public FX liabilities. Reserves, private foreign assets, market values, and fiscal expectations moved at the same time (Banco Central de la República Argentina, 2021; International Monetary Fund, 2018, 2019). The net effect is therefore ambiguous. We do not report δ because the available series cannot be placed on a common valuation and currency basis.

5.4. Japan: domestic-currency debt, public FX assets, and external wealth

Japan provides the opposite balance-sheet configuration. It combines very large domestic-currency public liabilities with substantial reserve assets and private external wealth.

At end-2025, central-government debt was JPY 1,342.2 trillion. The BOJ held JPY 544.4 trillion of government securities. It also reported JPY 120.6 trillion of banknotes, JPY 470.9 trillion of current deposits, and JPY 55.2 trillion of other deposits (Bank of Japan, 2026; Ministry of Finance Japan, 2026a). The last amount is a yen-denominated BOJ liability held by foreign central banks and others (Bank of Japan, 2025). We include it once in H and not in D_C^* . Government deposits are reported separately and excluded from H .

The resulting consolidated accounting benchmark is about JPY 1,444.6 trillion. It combines central-government and BOJ accounts. It is not an SNA general-government market-value balance sheet and contains no duration adjustment.

Official reserve assets totaled US\$1.370 trillion at end-2025. Of this amount, US\$1.164 trillion consisted of foreign-currency securities and deposits. The remaining assets were the IMF reserve position, SDRs, gold, and other reserve assets (Ministry of Finance Japan, 2026b,c). We use the foreign-currency component as the direct reserve-based counterpart to R^* and report the other components separately.

The reserve template also reports US\$13.6 billion of identified contingent short-term FX drains. These drains are liquidity indicators, not a complete measure of public FX liabilities. Foreign-currency reserves exceed the reported drains, so the reserve-based partial benchmark is nonnegative. The complete consolidated public FX position is not measured.

The calculation conditions on the fiscal-led valuation regime used in the theory. It does not classify Japan's actual monetary-fiscal regime.

Japan's private external wealth is large, but the aggregate data do not measure the private foreign-currency stock required by the model. Direct investment, securities, deposits, loans, and other claims differ in currency and fiscal accessibility. We therefore do not treat total private external assets as public backing or as a point estimate of A_{FX}^* .

The reserve-based partial result is informative even with this limitation. Its valuation schedule is increasing because observed foreign-currency reserves exceed reported contingent drains. This slope statement applies only to the reported reserve-based benchmark. The slope for the complete consolidated public balance sheet is not identified. Large domestic-currency liabilities can still transmit fiscal under-backing to the exchange rate through Propositions 1–3. Japan therefore separates that basic mechanism from the additional nonlinearity created by a negative public FX position.

5.5. Comparing Argentina and Japan

The two countries show how the additional fold mechanism depends on the measured public balance sheet. Domestic-currency debt creates the basic fiscal-depreciation mechanism in both cases. A fold requires the extra condition of a sufficiently negative public FX position.

Argentina supplies the mismatch case. Japan supplies a reserve-based partial comparison. The exercises also show why national external wealth cannot be treated as public backing: private assets matter only through

Table 7: Argentina and Japan: reported balance-sheet benchmarks

Question	Argentina	Japan
Observed balance-sheet components	Negative-position proxy	Foreign-currency reserves exceed reported contingent drains
Complete consolidated public $\mathcal{F}^*$	Not fully measured	Not fully measured
Slope in the reported benchmark	Negative at $\chi = 0$	Positive in the reserve-based partial benchmark
Slope for the complete public balance sheet	Not exactly identified	Not identified
Main role in the paper	Currency-mismatch illustration	Reserve-based nonnegative comparison
Main data limitation	Debt currency inferred partly from residency	Complete public FX liabilities unavailable

Note: The reported slopes refer to the stated benchmark calculations, not to complete estimates of the consolidated public-sector balance sheet. The country values are not directly comparable.

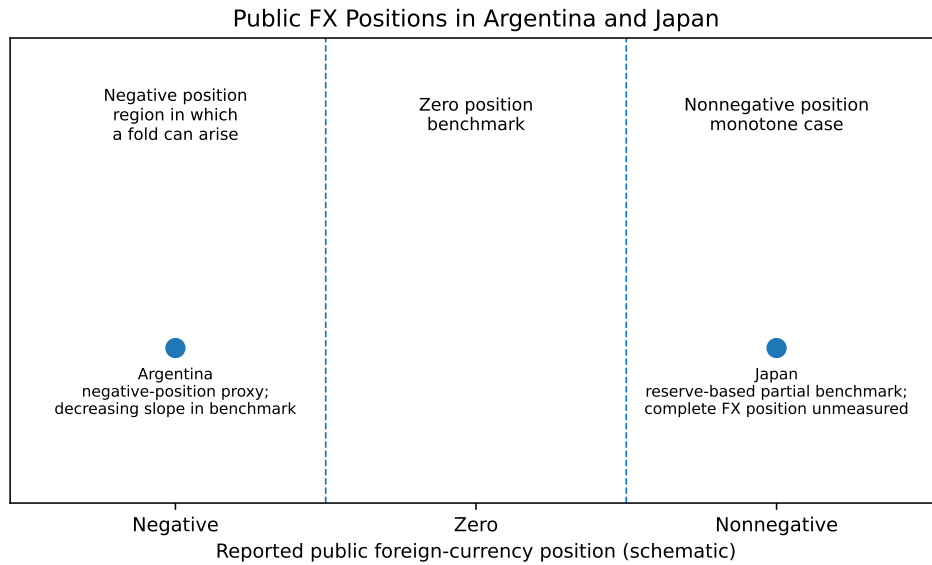


Figure 8: Public FX Positions in Argentina and Japan. The figure is schematic and categorical. The countries have different reference dates, data coverage, and valuation bases and are not placed on a common cardinal scale. Argentina has a negative-position proxy and a decreasing local slope in the fixed-backing benchmark. Japan has a nonnegative reserve-based partial benchmark; the complete consolidated public FX position and its slope are not measured.

their foreign-currency share and the present value of future tax revenue.

Neither country is a near-peak estimate based on security-level market values. Finding such an episode remains an important task for future research.

6. Conclusion

Depreciation has two opposing fiscal effects. It raises the CPI and lowers the real value of domestic-currency public debt. It also raises the domestic-currency value of foreign-currency public liabilities. Under the global constant-elasticity benchmark, a sufficiently negative public FX position makes the valuation schedule hump-shaped.

The generalized fold depends on $\gamma = \eta + \chi$. When $0 < \gamma < 1$, the ratio $r_\chi = \delta/\gamma$ identifies the local branch, $\chi^{\text{tan}} = \delta - \eta$ gives the backing elasticity that places the reference point at the peak, and d_{fold} gives a signed distance to that peak. Proposition 4 counts all mathematical solutions under the global benchmark. A local application admits only solutions inside the maintained pricing interval. The resulting multiplicity shows that the valuation relation alone need not be globally unique; it does not overturn determinacy results from complete monetary–fiscal regimes.

Argentina and Japan illustrate different balance sheets. With fixed backing, Argentina’s benchmark has r near 1.8 and lies on the decreasing branch. The classification reverses when χ exceeds about 0.20, but the exercise does not estimate χ or identify a historical branch. For Japan, foreign-currency reserves exceed reported contingent drains, so the reported reserve-based partial benchmark is nonnegative and increasing. The complete consolidated public FX positions and slopes are not measured for either country.

The model does not select a branch dynamically or explain default and peg exit. Future work should combine security-level currency decomposition, market values and duration, state-dependent pass-through, endogenous fiscal backing, and complete monetary–fiscal policy rules. Richer data can also identify whether any historical episode lies near the peak within a validated local pricing interval.

A. Sectoral valuation equations

A.1. Fiscal authority

The iteration uses the time-consistency property $\mathcal{M}_{t,t+j} = \mathcal{M}_{t,t+1}\mathcal{M}_{t+1,t+j}$ for the real stochastic discount factor.

The fiscal authority's recursive valuation relation implied by the intertemporal budget constraint and asset pricing is

$$\mathcal{L}_t^G = s_t^G + z_t + \mathbb{E}_t [\mathcal{M}_{t,t+1}\mathcal{L}_{t+1}^G]. \quad (113)$$

Iterating $J + 1$ periods yields

$$\mathcal{L}_t^G = \mathbb{E}_t \sum_{j=0}^J \mathcal{M}_{t,t+j}(s_{t+j}^G + z_{t+j}) + \mathbb{E}_t [\mathcal{M}_{t,t+J+1}\mathcal{L}_{t+J+1}^G]. \quad (114)$$

Impose the no-Ponzi condition

$$\lim_{J \rightarrow \infty} \mathbb{E}_t [\mathcal{M}_{t,t+J+1}\mathcal{L}_{t+J+1}^G] = 0. \quad (115)$$

Using eqs. (3) and (4) gives

$$\mathcal{L}_t^G = S_t^G + Z_t, \quad (116)$$

which, together with eq. (2), yields eq. (5).

A.2. Central bank

The central bank's recursive valuation relation implied by the intertemporal budget constraint and asset pricing is

$$\mathcal{L}_t^C = s_t^C - z_t + \mathbb{E}_t [\mathcal{M}_{t,t+1}\mathcal{L}_{t+1}^C]. \quad (117)$$

Iterating gives

$$\mathcal{L}_t^C = \mathbb{E}_t \sum_{j=0}^J \mathcal{M}_{t,t+j}(s_{t+j}^C - z_{t+j}) + \mathbb{E}_t [\mathcal{M}_{t,t+J+1}\mathcal{L}_{t+J+1}^C]. \quad (118)$$

Impose

$$\lim_{J \rightarrow \infty} \mathbb{E}_t [\mathcal{M}_{t,t+J+1}\mathcal{L}_{t+J+1}^C] = 0. \quad (119)$$

Then

$$\mathcal{L}_t^C = S_t^C - Z_t, \quad (120)$$

which, together with eq. (6), yields eq. (8).

A.3. Consolidation

Write the fiscal-authority equation as

$$\frac{B_{t-1}^G}{P_t} + \frac{E_t D_{t-1}^{G*}}{P_t} - Z_t = S_t^G + \frac{E_t R_{t-1}^{G*}}{P_t}, \quad (121)$$

and the central-bank equation as

$$\frac{H_{t-1}}{P_t} + \frac{E_t D_{t-1}^{C*}}{P_t} + Z_t = \frac{B_{t-1}^C}{P_t} + \frac{E_t R_{t-1}^{C*}}{P_t} + S_t^C. \quad (122)$$

Adding them and applying eqs. (9) to (12) gives

$$\frac{\mathcal{B}_{t-1}}{P_t} + \frac{E_t D_{t-1}^*}{P_t} = S_t + \frac{E_t R_{t-1}^*}{P_t}, \quad (123)$$

which is eq. (13).

B. Consumer-price index and effective pass-through

The household minimizes

$$P_{H,t} C_{H,t} + P_{M,t} C_{M,t} \quad (124)$$

subject to eq. (19). The first-order condition is

$$\frac{P_{H,t}}{P_{M,t}} = \frac{1-\alpha}{\alpha} \frac{C_{M,t}}{C_{H,t}}. \quad (125)$$

The expenditure shares are

$$P_{H,t} C_{H,t} = (1-\alpha) P_t C_t, \quad (126)$$

$$P_{M,t} C_{M,t} = \alpha P_t C_t. \quad (127)$$

Substitution into the aggregator yields

$$P_t = P_{H,t}^{1-\alpha} P_{M,t}^\alpha. \quad (128)$$

Using eq. (21),

$$P_t = P_{H,t}^{1-\alpha} [\Psi_t (E_t P_t^*)^\lambda]^\alpha \quad (129)$$

$$= P_{H,t}^{1-\alpha} \Psi_t^\alpha (P_t^*)^{\alpha\lambda} E_t^{\alpha\lambda} = \Omega_t E_t^\eta, \quad (130)$$

where $\eta = \alpha\lambda$ and Ω_t is defined in eq. (24).

C. Proofs

C.1. Proof of Proposition 1

With $\mathcal{F}_{t-1}^* = 0$, take logs of eq. (35):

$$\log \mathcal{B}_{t-1} = \log \Omega_t + \eta \log E_t + \log S_t(\mathcal{B}_{t-1}, E_t). \quad (131)$$

Differentiate along the local equilibrium while holding Ω_t fixed:

$$1 = \eta \frac{d \log E_t}{d \log \mathcal{B}_{t-1}} + \xi_t + \chi_t \frac{d \log E_t}{d \log \mathcal{B}_{t-1}}. \quad (132)$$

Rearranging yields eq. (36). The sign statements follow when $\eta + \chi_t > 0$. Setting $\chi_t = 0$ gives eq. (38); setting additionally $\xi_t = 0$ and solving the level equation gives eq. (39). $\square$

C.2. Proof of Proposition 2

Let

$$V_t(E) = S_t \Omega_t E^\eta + \mathcal{F}_{t-1}^* E. \quad (133)$$

Then $V_t(0) = 0$, $\lim_{E \rightarrow \infty} V_t(E) = \infty$, and

$$V'_t(E) = \eta S_t \Omega_t E^{\eta-1} + \mathcal{F}_{t-1}^* > 0. \quad (134)$$

Hence a unique positive solution exists for every $\mathcal{B}_{t-1} > 0$. Implicit differentiation gives eqs. (42) to (44). $\square$

C.3. Proof of Proposition 3

Differentiate eq. (25) in logarithms with respect to $\mathcal{B}_{t-1}$, holding Ω_t and $\mathcal{F}_{t-1}^*$ fixed. Define the local elasticity as $\varepsilon_t \equiv d \log E_t / d \log \mathcal{B}_{t-1}$. The shares in eqs. (45) and (46) then imply

$$1 = \omega_{S,t} [\xi_t + (\eta + \chi_t) \varepsilon_t] + \omega_{F,t} \varepsilon_t. \quad (135)$$

Solving gives eq. (48). For $\chi_t = 0$, use $\omega_{S,t} = 1 - \omega_{F,t}$ to write

$$\varepsilon_t = \frac{1 - (1 - \omega_{F,t}) \xi_t}{\eta(1 - \omega_{F,t}) + \omega_{F,t}}. \quad (136)$$

Differentiation with respect to $\omega_{F,t}$ yields

$$\frac{\partial \varepsilon_t}{\partial \omega_{F,t}} = \frac{\eta + \xi_t - 1}{\Lambda_t^2}, \quad (137)$$

which proves eqs. (51) and (52). Replacing η by $\eta + \chi_t$ gives eq. (53). $\square$

C.4. Proof of Corollary 2

From eq. (17), $\partial \mathcal{F}_{t-1}^* / \partial A_{t-1}^* = \theta$. Applying the chain rule at a locally regular equilibrium gives eq. (54). $\square$

C.5. Proof of Proposition 4

Define

$$G_t(E) \equiv S_t \Omega_t E^\eta - \mathcal{D}_{t-1}^* E. \quad (138)$$

For $0 < \eta < 1$,

$$G'_t(E) = \eta S_t \Omega_t E^{\eta-1} - \mathcal{D}_{t-1}^*, \quad (139)$$

$$G''_t(E) = \eta(\eta - 1) S_t \Omega_t E^{\eta-2} < 0. \quad (140)$$

The unique maximizer solves $G'_t(E) = 0$ and is eq. (64). At this point,

$$\mathcal{D}_{t-1}^* E_t^\dagger = \eta S_t \Omega_t (E_t^\dagger)^\eta. \quad (141)$$

Therefore,

$$G_t(E_t^\dagger) = (1 - \eta) S_t \Omega_t (E_t^\dagger)^\eta \quad (142)$$

$$= (1 - \eta) S_t \Omega_t \left(\frac{\eta S_t \Omega_t}{\mathcal{D}_{t-1}^*} \right)^{\eta/(1-\eta)} = \overline{\mathcal{B}}_t^V. \quad (143)$$

Because $G_t(0) = 0$, G_t rises to its unique maximum and then tends to $-\infty$, the stated number of positive intersections follows. Equivalently, for $H(E, \mathcal{B}) = G_t(E) - \mathcal{B}$, the fold satisfies $H = H_E = 0$, while

$H_{EE} = \eta(\eta - 1)S_t\Omega_t(E_t^\dagger)^{\eta-2} \neq 0$ and $H_{\mathcal{B}} = -1 \neq 0$. These are the standard nondegeneracy conditions for a smooth fold. $\square$

C.6. Proof of Corollary 3

Let $A_t = S_t\Omega_t$ and write $D = \mathcal{D}_{t-1}^*$. At $D = 0$, the equation $\mathcal{B}_{t-1} = A_tE^\eta$ has the unique finite solution E_t^0 in eq. (67). The valuation bound satisfies

$$\overline{\mathcal{B}}_t^V(D) = (1 - \eta)A_t \left(\frac{\eta A_t}{D} \right)^{\eta/(1-\eta)} \longrightarrow \infty \quad (D \downarrow 0). \quad (144)$$

Hence, for fixed $\mathcal{B}_{t-1} > 0$, two positive solutions exist for every sufficiently small $D > 0$. Since the derivative of $A_tE^\eta - DE - \mathcal{B}_{t-1}$ with respect to E is strictly positive at $(E_t^0, 0)$, the implicit-function theorem gives a unique local solution continuous in D ; this is the increasing branch, and it converges to E_t^0 as $D \downarrow 0$.

For every $D > 0$ with two roots, the upper root exceeds the turning point

$$E_t^\dagger(D) = \left(\frac{\eta A_t}{D} \right)^{1/(1-\eta)}. \quad (145)$$

Because $E_t^\dagger(D) \rightarrow \infty$ as $D \downarrow 0$, the upper root also diverges. This proves eqs. (68) and (69). For completeness, fixing $\mathcal{B}_{t-1}$, the two branches meet when

$$E_t^c = \left[\frac{\mathcal{B}_{t-1}}{(1 - \eta)A_t} \right]^{1/\eta}, \quad \overline{D}_t^* = \eta A_t (E_t^c)^{\eta-1}. \quad (146)$$

As D rises to $\overline{D}_t^*$, the two roots coalesce at E_t^c ; for larger D , no positive exchange rate satisfies the maintained date- t valuation equation. $\square$

C.7. Proof of Proposition 5

Under eq. (72), define $\gamma = \eta + \chi$ and

$$G_t^\chi(E) = \bar{S}_t\Omega_t E^\gamma - \mathcal{D}_{t-1}^* E. \quad (147)$$

If $\gamma < 0$, then

$$(G_t^\chi)'(E) = \gamma \bar{S}_t\Omega_t E^{\gamma-1} - \mathcal{D}_{t-1}^* < 0, \quad (148)$$

so the valuation function is strictly decreasing and cannot have the fold in Proposition 4. If $\gamma = 0$, then $G_t^\chi(E) = \bar{S}_t\Omega_t - \mathcal{D}_{t-1}^* E$, which is linear and strictly decreasing. If $0 < \gamma < 1$, then

$$(G_t^\chi)''(E) = \gamma(\gamma - 1)\bar{S}_t\Omega_t E^{\gamma-2} < 0, \quad (149)$$

and the proof of Proposition 4 applies after replacing S_t by $\bar{S}_t$ and η by γ . If $\gamma = 1$, the valuation function is linear. If $\gamma > 1$, it is strictly convex. Hence the strict concave two-branch fold occurs if and only if $0 < \gamma < 1$. $\square$

C.8. Proof of Proposition 6

If the real exchange rate is fixed at Q_t , then eq. (1) implies

$$P_t = \frac{E_t P_t^*}{Q_t}. \quad (150)$$

Substitute this expression and $\mathcal{F}_{t-1}^* = -\mathcal{D}_{t-1}^*$ into eq. (18):

$$\mathcal{B}_{t-1} = \frac{E_t P_t^*}{Q_t} S_t - E_t \mathcal{D}_{t-1}^* \quad (151)$$

$$= E_t \left(\frac{P_t^* S_t}{Q_t} - \mathcal{D}_{t-1}^* \right). \quad (152)$$

The right-hand side is linear in E_t . It has a unique positive solution if $P_t^* S_t > Q_t \mathcal{D}_{t-1}^*$ and, if this condition fails, no positive exchange rate satisfies the maintained date- t valuation equation for $\mathcal{B}_{t-1} > 0$. $\square$

D. Long-duration public liabilities

Suppose the fiscal authority has a portfolio of home-currency nominal liabilities indexed by maturity j , with promised nominal payment X_{t+j} and date- t nominal price $Q_{t,j}^B$. The inherited market value is

$$B_{t-1}^{G,mv} \equiv \sum_{j \geq 0} Q_{t,j}^B X_{t+j}. \quad (153)$$

After subtracting the market value held by the central bank and adding monetary liabilities, define

$$\mathcal{B}_{t-1}^{mv} \equiv B_{t-1}^{G,mv} - B_{t-1}^{C,mv} + H_{t-1}. \quad (154)$$

The valuation identity continues to hold conditionally on this inherited market value.

The propositions continue to hold conditionally on the inherited market value of public liabilities. If fiscal news simultaneously changes bond prices, the market value of outstanding liabilities becomes endogenous and a term-structure model is required for full comparative statics. Market-value accounting preserves the identity, but it does not imply that every comparative static derived while holding $\mathcal{B}_{t-1}$ fixed remains valid when fiscal news reprices the term structure. Maturity affects the conditional exercise through the inherited market value of nominal liabilities. When fiscal or monetary news reprices the term structure, maturity also affects the mapping from promised nominal payments to that market value. Modeling this repricing requires an additional asset-pricing block for yields, duration, and risk premia and lies outside the present date- t exercise.

E. National resources versus fiscal backing

Let L_{t-1}^* denote private foreign liabilities. National net foreign assets, measured as nominal amounts in foreign currency, are

$$N_{t-1}^* \equiv A_{t-1}^* - L_{t-1}^* + R_{t-1}^* - D_{t-1}^*. \quad (155)$$

In a terminal-period representation with home output Y_t , government purchases G_t , and private consumption C_t , consolidating domestic transfers gives

$$P_t (C_t + G_t) = P_{H,t} Y_t + E_t N_{t-1}^*. \quad (156)$$

The tax-claim coefficient θ does not appear because taxes on private foreign wealth are domestic transfers. The national resource identity depends on the country's total external position, while the public valuation equation depends on the narrower set of foreign-currency resources available to the consolidated public sector. This distinction prevents national external wealth from being treated as public backing at full value.

F. Argentina data appendix

The Argentina application uses a contractual-exposure stress screening because a common inherited market-value, currency, issuer, and holder decomposition is not available. The public external-debt series is residency based. It can omit domestically held foreign-currency obligations and include nonresident-held domestic-currency obligations, so its mapping error relative to D_{gross}^* is potentially two sided. The residual of total public debt less external debt is consequently labeled a residual nonexternal-debt proxy, not consolidated domestic-currency public liabilities.

The private nonfinancial external-asset stock is also broader than the model's private foreign-currency asset. The sensitivity relation $A_{\text{FX}}^* = \kappa_A A_{\text{external}}$ makes that mapping explicit. The aggregate external-asset proxy separately measures neither θ nor κ_A ; only the product $\tilde{\theta} = \theta\kappa_A$ enters the calculation. The reported scenarios use $\kappa_A \in \{0.25, 0.50, 0.75, 1.00\}$ and $\theta \in \{0, 0.05, 0.10, 0.25, 0.50\}$.

Table A1: Argentina: Pass-Through Sensitivity

Year	Effective CPI pass-through η						
	0.10	0.15	0.20	0.25	0.30	0.40	0.50
2000: $r = \delta/\eta$	4.554	3.036	2.277	1.822	1.518	1.139	0.911
2001: $r = \delta/\eta$	4.513	3.009	2.257	1.805	1.504	1.128	0.903

Note: The fixed-public-resource benchmark uses $\chi = 0$, $\kappa_A = 1$, and $\kappa_R = 1$. The tangency pass-through threshold equals $\eta^{\text{tan}} = \delta$. With no tax claim on private foreign assets, it is about 0.455 in 2000 and 0.451 in 2001. Values below the threshold put the reference point on the decreasing part of the schedule; values above it put the reference point on the increasing part.

Table A2: Argentina: exchange-rate elasticity of fiscal backing

Year	χ	$\gamma = \eta + \chi$	δ	$r_\chi = \delta/\gamma$	d_{fold}	Local slope
2000	-0.20	0.05	0.455	9.108	-2.325	Decreasing
2000	0.00	0.25	0.455	1.822	-0.800	Decreasing
2000	0.10	0.35	0.455	1.301	-0.405	Decreasing
2000	0.20	0.45	0.455	1.012	-0.022	Near peak (decreasing)
2000	0.30	0.55	0.455	0.828	0.419	Increasing
2001	-0.20	0.05	0.451	9.026	-2.316	Decreasing
2001	0.00	0.25	0.451	1.805	-0.788	Decreasing
2001	0.10	0.35	0.451	1.289	-0.391	Decreasing
2001	0.20	0.45	0.451	1.003	-0.005	Near peak (decreasing)
2001	0.30	0.55	0.451	0.821	0.440	Increasing

Note: $\eta = 0.25$, $\theta = 0$, and $\kappa_A = \kappa_R = 1$. The fold diagnostic applies because every reported γ lies in $(0, 1)$. The peak thresholds are $\chi^{\text{tan}} = 0.205$ in 2000 and 0.201 in 2001. The calculation does not estimate χ .

The other public external-asset series is also broader than a measured foreign-currency asset stock. The auxiliary mapping $R_{\text{FX}}^* = R_{\text{reserve}}^* + \kappa_R R_{\text{other external}}^*$ therefore uses $\kappa_R \in \{0, 0.5, 1\}$. The baseline $\kappa_R = 1$ treats all reported other public external assets as foreign-currency assets. Lower values reduce measured public FX backing, raise the effective net FX liability and normalized proximity, and increase both tax-claim thresholds.

The mid-2001 calculation uses official reserve observations but no fabricated mid-year debt point. The 2018–19 discussion uses IMF and BCRA evidence on foreign-currency borrowing, reserves, and private foreign-asset accumulation, but no δ is reported because the available series do not share a common valuation basis or currency decomposition.

Table A3: Argentina: tax-claim thresholds under private-asset currency-share scenarios

Year	κ_A	$\theta^{\tan}$	θ^{crit}	Tangency feasible in $[0, 1]$	
2000	0.25	1.212	2.015	Yes / sign threshold marginally above one	No
2000	0.50	0.606	1.008		Yes
2000	0.75	0.404	0.672		Yes
2000	1.00	0.303	0.504		Yes
2001	0.25	1.528	2.569	Yes / sign threshold above one	No
2001	0.50	0.764	1.285		Yes
2001	0.75	0.509	0.856		Yes
2001	1.00	0.382	0.642		Yes

Note: The thresholds use the reported balance-sheet components, $\eta = 0.25$, $\chi = 0$, $\kappa_R = 1$, and a fixed reference exchange rate. $\theta = \theta^{\tan}$ places the reference point at the peak; a strict exceedance places it on the increasing side.

Table A4: Argentina: Public-Asset Currency-Share Sensitivity

Year	κ_R	R_{FX}^*	$\mathcal{D}^*(0)$	$\delta(0)$	$r(0)$	$\theta^{\tan}$	θ^{crit}
2000	0.0	26.917	58.026	0.493	1.974	0.386	0.587
2000	0.5	31.027	53.917	0.475	1.901	0.345	0.545
2000	1.0	35.136	49.807	0.455	1.822	0.303	0.504
2001	0.0	14.913	73.337	0.480	1.919	0.460	0.720
2001	0.5	18.867	69.384	0.466	1.863	0.421	0.681
2001	1.0	22.820	65.430	0.451	1.805	0.382	0.642

Note: Fixed-public-resource benchmark with $\chi = 0$, $\eta = 0.25$, $\theta = 0$, $\kappa_A = 1$, and the reported contractual-exposure perimeter. The table is a data-mapping sensitivity, not a new theoretical parameterization.

Table A5: Argentina: Currency-Reclassification Sensitivity

Year	Reclassification	Adjusted FX proxy	Residual proxy	r	Local classification
2000	-20%	67.95	76.55	1.200	Decreasing branch
2000	-10%	76.45	68.05	1.511	Decreasing branch
2000	0%	84.94	59.56	1.822	Decreasing branch
2000	+10%	93.44	51.06	2.132	Decreasing branch
2000	+20%	101.93	42.57	2.443	Decreasing branch
2001	-20%	70.60	97.20	1.318	Decreasing branch
2001	-10%	79.43	88.38	1.562	Decreasing branch
2001	0%	88.25	79.55	1.805	Decreasing branch
2001	+10%	97.08	70.73	2.049	Decreasing branch
2001	+20%	105.90	61.90	2.292	Decreasing branch

Note: The benchmark uses $\chi = 0$, $\eta = 0.25$, $\kappa_A = 1$, and $\kappa_R = 1$. Positive reclassification represents omitted domestically held FX debt; negative reclassification represents nonresident-held domestic-currency debt included in the external-debt proxy. Total reported public debt is held fixed. The exercise visualizes mapping uncertainty and is not a security-level reconstruction.

G. Japan data appendix

The Japan benchmark consolidates central-government debt with BOJ holdings and nominal BOJ liabilities held outside the fiscal–central-bank perimeter. It does not claim equivalence with the SNA general-government perimeter, and it does not convert promised payments into a duration-adjusted market value. The benchmark removes BOJ-held government securities and adds banknotes, current deposits, and other deposits. Deposits of the government are separately reported and excluded, as are other identifiable intrapublic positions. The JPY 55.2 trillion item is the BOJ “other deposits” balance-sheet line, not a second external-deposit stock. The BOJ identifies it as deposits held by foreign central banks and others, and its market-operation reports describe the relevant balances as yen-denominated deposits. It is therefore included once in H and not entered in D_C^* . This treatment is conditional on the fiscal-led valuation regime in the paper and is not an empirical classification of Japan’s monetary–fiscal policy regime.

The end-2025 reserve template reports total official reserve assets of US\$1.370 trillion, comprising US\$1.164 trillion of foreign-currency reserves, an IMF reserve position of US\$11.3 billion, SDRs of US\$60.8 billion, gold of US\$117.2 billion, and other reserve assets of US\$16.3 billion. The direct reserve-based counterpart to model R^* is the foreign-currency reserve component. Gold, SDRs, the IMF reserve position, and other components are disclosed separately because they need not revalue one for one against the selected foreign currency. The identified contingent short-term FX drains are also reported separately and are not equated with the complete stock of gross public FX liabilities.

Total private external assets are not entered as A_{FX}^* without a currency-denomination, ownership, and tax-claim mapping. Japan-specific effective CPI pass-through is reported as an illustrative range informed by evidence on import-price and consumer-price transmission rather than imported from the Argentina scenarios.

Table A6: Japan: public-balance-sheet and reserve-based public-FX benchmark, end-2025

Object	Statistical perimeter	Value	Treatment
Central-government debt	MOF central government	JPY 1,342.2tn	Accounting/face; differs from SNA general government
BOJ government securities	BOJ asset	JPY 544.4tn	Removed in consolidation
Banknotes and current deposits	Outside-perimeter	JPY 591.6tn	Yen liabilities included in H
BOJ other deposits	BOJ liabilities		
	Foreign central banks and others	JPY 55.2tn	Same yen-denominated stock previously listed as an external-deposit proxy; included once in H , not in D_C^*
Deposits of the government	Intrapublic BOJ liability	JPY 5.1tn	Reported separately and excluded from H
Consolidated accounting benchmark	Constructed	JPY 1,444.6tn	No duration or term-structure adjustment
Total official reserve assets	Reserve template	US\$1,369.8bn	Broader reserve total; not equated one for one with model R^*
Foreign-currency reserves	Reserve template	US\$1,164.2bn	Direct reserve-based counterpart to model R^*
Identified contingent short-term FX drains	Reserve template	US\$13.6bn	Liquidity indicator, not complete public FX liability stock
Private external assets	Nonpublic external sectors	Not used as point input	Currency-denominated fiscal stock not separately measured
Effective CPI pass-through	Scenario range	0.08–0.20	Illustrative Japan-specific range

H. Institutional, valuation-basis, and consolidation checks

Table A7: Japan: mapping official reserve components to model foreign-currency backing

Reserve component	End-2025 value	Direct model treatment	Measurement interpretation
Foreign-currency reserves	US\$1,164.196bn	Included in reserve-based R^*	Securities and deposits denominated in foreign currency
IMF reserve position	US\$11.262bn	Reported separately	Multilateral reserve claim; not assumed to move one for one with a selected bilateral exchange rate
SDRs	US\$60.840bn	Reported separately	Basket-denominated reserve asset
Gold	US\$117.172bn	Reported separately	Market-valued noncurrency reserve asset
Other reserve assets	US\$16.305bn	Reported separately	Requires instrument-specific mapping
Total official reserve assets	US\$1,369.775bn	Not used mechanically as model R^*	Sum of heterogeneous reserve components

Note: The same principle applies to Argentina: official reserves are a reserve-based foreign-asset benchmark, but components that do not revalue one for one against the selected foreign currency are not exact model R^* without an additional mapping.

Table A8: Institutional and valuation mapping

Country	Series	Issuer perimeter	Holder or residency basis	Currency basis	Valuation basis
Argentina	Total public debt	IMF-reported public sector	Mixed	Mixed	Contractual/face
Argentina	Public external-debt proxy	Reported public sector/BCRA	Nonresident-creditor basis	Currency split unavailable	Reported book/residual nominal
Argentina	Official reserves	BCRA	External asset	Reserve currencies	Official reserve basis
Argentina	Other public external assets	Public-sector/BCRA mapping	External asset	Mapped with κ_R ; not fully decomposed	Reported book
Argentina	Private external assets	Private nonfinancial sector	External asset	External-asset proxy mapped with κ_A	IIP basis
Japan	Central-government debt	Central government	All holders	Predominantly yen; instrument-level split unavailable	Accounting/face
Japan	BOJ government securities	BOJ asset and central-government liability	BOJ	Yen	BOJ accounts
Japan	Banknotes and current deposits	BOJ liabilities outside consolidated perimeter	Financial institutions and other outside holders	Yen liabilities	BOJ accounts
Japan	BOJ other deposits	BOJ liability outside consolidated perimeter	Foreign central banks and others	Yen-denominated deposits; same stock included once in H	BOJ accounts
Japan	Government deposits	BOJ liability to fiscal authority	Intrapublic	Yen	Excluded in consolidation
Japan	Foreign-currency reserves	Reserve perimeter	External asset	Foreign currency	Direct reserve-based model- R^* benchmark
Japan	Other official reserve components	Reserve perimeter	External asset	IMF claim, SDR basket, gold, and other	Reported separately; not mechanically equated to model R^*
Japan	Identified contingent short-term FX drains	Reserve-template public perimeter	Contingent liquidity claims	Foreign currency	Indicator, not complete FX liability stock
Japan	Private external assets	Nonpublic external sectors	External asset	Not fully identified	IIP basis

Table A9: Consistency of valuation exercises

Exercise	Permitted use	Restriction
Observed market value	Closest counterpart to inherited market value in the theory	Includes expectations of default, exit, liquidity, and haircuts; requires a common currency split
Pre-distress market value	Counterfactual sensitivity before acute repricing	Must be labeled counterfactual and dated consistently
Contractual face value	Illustrative exercise based on contractual exposures	Not a market-value equilibrium calculation
Japan accounting benchmark	Consolidated stock mapping	Does not model duration or term-structure repricing

Checks: central-bank-held government debt is not double counted; gross reserves are separated from contingent drains; residency is not treated as currency denomination; private external assets are not treated as public assets; other public external assets are mapped with κ_R ; government deposits are excluded from Japan's consolidated H ; the JPY 55.2 trillion BOJ other-deposits stock is included once in H and not in D_C^* ; official reserve components are mapped separately; and no reported calculation combines market-value domestic liabilities with face-value FX positions.

References

- Aghion, Philippe, Philippe Bacchetta, and Abhijit Banerjee (2001). “Currency Crises and Monetary Policy in an Economy with Credit Constraints”. In: *European Economic Review* 45.7, pp. 1121–1150.
- Aguiar, Mark and Gita Gopinath (2006). “Defaultable Debt, Interest Rates and the Current Account”. In: *Journal of International Economics* 69.1, pp. 64–83.
- Arellano, Cristina (2008). “Default Risk and Income Fluctuations in Emerging Economies”. In: *American Economic Review* 98.3, pp. 690–712.
- Banco Central de la República Argentina (2021). *Foreign Exchange Market, Debt and Build-Up of Foreign Assets, 2015–2019*. BCRA.
- Bank of Japan (2025). *Market Operations in Fiscal 2024*. Bank of Japan.
- (2026). *Bank of Japan Accounts, December 31, 2025*. Bank of Japan.
- Bassetto, Marco and Todd Messer (2013). “Fiscal Consequences of Paying Interest on Reserves”. In: *Fiscal Studies* 34.4, pp. 413–436.
- Bénétrix, Agustín S., Philip R. Lane, and Jay C. Shambaugh (2015). “International Currency Exposures, Valuation Effects and the Global Financial Crisis”. In: *Journal of International Economics* 96.S1, S98–S109.
- Bergin, Paul R. (2000). “Fiscal Solvency and Price Level Determination in a Monetary Union”. In: *Journal of Monetary Economics* 45.1, pp. 37–53.
- Bianchi, Francesco, Renato Faccini, and Leonardo Melosi (2023). “A Fiscal Theory of Persistent Inflation”. In: *Quarterly Journal of Economics* 138.4, pp. 2127–2179.
- Buiter, Willem H. (2002). “The Fiscal Theory of the Price Level: A Critique”. In: *Economic Journal* 112.481, pp. 459–480.
- Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo (2001). “Prospective Deficits and the Asian Currency Crisis”. In: *Journal of Political Economy* 109.6, pp. 1155–1197.
- Caselli, Francesca and Agustin Roitman (2016). *Non-Linear Exchange Rate Pass-Through in Emerging Markets*. IMF Working Paper 16/1. International Monetary Fund.
- Cavallino, Paolo (2019). “Capital Flows and Foreign Exchange Intervention”. In: *American Economic Journal: Macroeconomics* 11.2, pp. 127–170.
- Céspedes, Luis Felipe, Roberto Chang, and Andrés Velasco (2004). “Balance Sheets and Exchange Rate Policy”. In: *American Economic Review* 94.4, pp. 1183–1193.
- Cochrane, John H. (2001). “Long-Term Debt and Optimal Policy in the Fiscal Theory of the Price Level”. In: *Econometrica* 69.1, pp. 69–116.
- (2023). *The Fiscal Theory of the Price Level*. Princeton, NJ: Princeton University Press.
- Corsetti, Giancarlo and Bartosz Maćkowiak (2006). “Fiscal Imbalances and the Dynamics of Currency Crises”. In: *European Economic Review* 50.5, pp. 1317–1338.
- Daniel, Betty C. (2001). “The Fiscal Theory of the Price Level in an Open Economy”. In: *Journal of Monetary Economics* 48.2, pp. 293–308.
- Daseking, Christina, Atish Ghosh, Timothy Lane, and Steven Phillips (2005). *Lessons from the Crisis in Argentina*. International Monetary Fund.
- Del Negro, Marco and Christopher A. Sims (2015). “When Does a Central Bank’s Balance Sheet Require Fiscal Support?” In: *Journal of Monetary Economics* 73, pp. 1–19.
- Du, Wenxin and Jesse Schreger (2016). “Local Currency Sovereign Risk”. In: *Journal of Finance* 71.3, pp. 1027–1070.
- Dupor, Bill (2000). “Exchange Rates and the Fiscal Theory of the Price Level”. In: *Journal of Monetary Economics* 45.3, pp. 613–630.

- Eichengreen, Barry, Ricardo Hausmann, and Ugo Panizza (2003). *Currency Mismatches, Debt Intolerance and Original Sin: Why They Are Not the Same and Why It Matters*. NBER Working Paper 10036. National Bureau of Economic Research.
- Evans, Martin D. D. and Richard K. Lyons (2002). “Order Flow and Exchange Rate Dynamics”. In: *Journal of Political Economy* 110.1, pp. 170–180.
- Gabaix, Xavier and Matteo Maggiori (2015). “International Liquidity and Exchange Rate Dynamics”. In: *Quarterly Journal of Economics* 130.3, pp. 1369–1420.
- Gali, Jordi and Tommaso Monacelli (2005). “Monetary Policy and Exchange Rate Volatility in a Small Open Economy”. In: *Review of Economic Studies* 72.3, pp. 707–734.
- Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon (2010). “Currency Choice and Exchange Rate Pass-Through”. In: *American Economic Review* 100.1, pp. 304–336.
- Gourinchas, Pierre-Olivier and Hélène Rey (2007). “International Financial Adjustment”. In: *Journal of Political Economy* 115.4, pp. 665–703.
- Hall, Robert E. and Ricardo Reis (2015). *Maintaining Central-Bank Financial Stability under New-Style Central Banking*. NBER Working Paper 21173. National Bureau of Economic Research.
- Hilscher, Jens, Alon Raviv, and Ricardo Reis (2022). “Inflating Away the Public Debt? An Empirical Assessment”. In: *Review of Financial Studies* 35.3, pp. 1553–1595.
- Independent Evaluation Office of the International Monetary Fund (2004). *The IMF and Argentina, 1991–2001*. International Monetary Fund.
- Instituto Nacional de Estadística y Censos (2001). *La Posición de Inversión Internacional de Argentina a Fines del Año 2000*. Ministerio de Economía, República Argentina.
- (2002). *La Posición de Inversión Internacional de Argentina a Fines del Año 2001*. Ministerio de Economía, República Argentina.
- International Monetary Fund (2004). *Argentina: Second Review under the Stand-By Arrangement*. IMF Country Report 04/195. International Monetary Fund.
- (2018). *Argentina: Request for Stand-By Arrangement*. IMF Country Report 18/219. International Monetary Fund.
- (2019). *Argentina: Third Review under the Stand-By Arrangement*. IMF Country Report 19/99. International Monetary Fund.
- Jiang, Zhengyang (2022). “Fiscal Cyclicalities and Currency Risk Premia”. In: *The Review of Financial Studies* 35.3, pp. 1527–1552.
- Jiang, Zhengyang, Hanno Lustig, Stijn Van Nieuwerburgh, and Mindy Z. Xiaolan (2024). “The U.S. Public Debt Valuation Puzzle”. In: *Econometrica* 92.4, pp. 1309–1347.
- Krugman, Paul R. (1988). “Financing vs. Forgiving a Debt Overhang”. In: *Journal of Development Economics* 29.3, pp. 253–268.
- (1999). “Balance Sheets, the Transfer Problem, and Financial Crises”. In: *International Tax and Public Finance* 6.4, pp. 459–472.
- Lane, Philip R. and Jay C. Shambaugh (2010). “Financial Exchange Rates and International Currency Exposures”. In: *American Economic Review* 100.1, pp. 518–540.
- Leeper, Eric M. (1991). “Equilibria under ‘Active’ and ‘Passive’ Monetary and Fiscal Policies”. In: *Journal of Monetary Economics* 27.1, pp. 129–147.
- McCallum, Bennett T. (2001). “Indeterminacy, Bubbles, and the Fiscal Theory of Price Level Determination”. In: *Journal of Monetary Economics* 47.1, pp. 19–30.
- Ministry of Finance Japan (2026a). *Central Government Debt as of the End of December 2025*. Ministry of Finance Japan.
- (2026b). *International Investment Position of Japan, End of 2025*. Ministry of Finance Japan.
- (2026c). *International Reserves and Foreign Currency Liquidity, End of December 2025*. Ministry of Finance Japan.

- Monacelli, Tommaso and Roberto Perotti (2010). “Fiscal Policy, the Real Exchange Rate and Traded Goods”. In: *Economic Journal* 120.544, pp. 437–461.
- Obstfeld, Maurice and Kenneth Rogoff (1995). “Exchange Rate Dynamics Redux”. In: *Journal of Political Economy* 103.3, pp. 624–660.
- Ravn, Morten O., Stephanie Schmitt-Grohé, and Martín Uribe (2012). “Consumption, Government Spending, and the Real Exchange Rate”. In: *Journal of Monetary Economics* 59.3, pp. 215–234.
- Reis, Ricardo (2015). *Different Types of Central Bank Insolvency and the Central Role of Seignorage*. NBER Working Paper 21226. National Bureau of Economic Research.
- (2016). *Can the Central Bank Alleviate Fiscal Burdens?* NBER Working Paper 23014. National Bureau of Economic Research.
- Sargent, Thomas J. and Neil Wallace (1981). “Some Unpleasant Monetarist Arithmetic”. In: *Federal Reserve Bank of Minneapolis Quarterly Review* 5.3, pp. 1–17.
- Sarno, Lucio and Mark P. Taylor (2001). “Official Intervention in the Foreign Exchange Market: Is It Effective and, If So, How Does It Work?” In: *Journal of Economic Literature* 39.3, pp. 839–868.
- Sims, Christopher A. (1994). “A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy”. In: *Economic Theory* 4.3, pp. 381–399.
- Sturzenegger, Federico and Jeromin Zettelmeyer (2006). *Debt Defaults and Lessons from a Decade of Crises*. Cambridge, MA: MIT Press.
- Woodford, Michael (1995). “Price-Level Determinacy without Control of a Monetary Aggregate”. In: *Carnegie-Rochester Conference Series on Public Policy* 43, pp. 1–46.